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# An Agent-Based Modeling Approach to Determine Winter Survival Rates of American Robins and Eastern Bluebirds

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## Abstract

American Robins (*Turdus migratorius*) and Eastern Bluebirds (*Sialia sialis*) are two species of migratory thrushes that breed in Northwest Indiana but historically are uncommon during the winter season. These trends have changed recently, and both species are seen more abundantly during the winter. Recently invaded non-native fruiting plants continue to provide nutrients for the birds throughout the winter and may contribute to the increased avian populations during that time. To measure the effect these food sources contribute to thrush wintering habits, we created an agent-based computer model to simulate the birds' movement in Northwest Indiana along with their food consumption over the course of the winter season. The model incorporates availability of food sources, foraging and roosting behavior, bio-energetics, and starvation, with parameter values informed by the literature. We obtained simulated winter survival rates of the birds that could begin to explain the changes in the birds' migratory patterns.

**Keywords:** agent-based model, American robin, eastern bluebird, population dynamics, invasive species, migration, phenology

## 1 Introduction

American Robins (*Turdus migratorius*) and Eastern Bluebirds (*Sialia sialis*) are common migratory birds in Northwest Indiana. In nearby Iowa, it has been found that although most Eastern Bluebirds migrated for the winter, some of the birds remained and depended on fleshy, invasive fruits as a food source [7]. With a comparable winter climate, we have focused this project in Northwest Indiana, where there are a number of invasive fruiting plants that continue to provide nourishment to the birds over the course of the winter months, some of which include Autumn Olive (*Elaeagnus umbellata*), Amur Bush Honeysuckle (*Lonicera maackii*), Multiflora Rose (*Rosa multiflora*), Highbush Cranberry (*Viburnum opulus*), Oriental Bittersweet (*Celastrus orbiculatus*), and Bittersweet Nightshade (*Solanum dulcamara*) [1, 6]. Across the nation, the flowering periods of these species vary. In Illinois, the blooming season of Oriental Bittersweet begins in early May [8]; in even more northern climes like Vermont, the plant is usually in full bloom and has fruited by late October with fruits staying on the vine well into winter [10]. Research shows that these surviving plants, then, remain a food source for American Robins and Eastern Bluebirds alike over the winter months [5].

This change in resources raises the question of what

is the winter survival rate of these birds. If the presence of additional food sources significantly increases the likelihood of survival of the birds, it could imply that these invasive plants are a significant contributor to the thrushes' newfound tendency to overwinter in Northwest Indiana. To determine the survival rate, factors such as basal metabolic rate (BMR), foraging patterns, roosting behavior, realistic range sizes, and different habitats were taken into consideration to create an agent-based model (ABM).

ABMs are a helpful tool that are often used to model and determine various emergent patterns or behaviors when dealing with populations over time [11]. The agent-based modeling technique provides a well-suited structure to model foraging and roosting patterns for birds, as it can model an entire bird population's movements and behavior, and at the same time track and record the properties of a single bird within the flock. ABMs are flexible and provide an easy way to manipulate population and resource sizes, thus allowing for simulation of specific environments.

In this project, we used Netlogo v. 5.2 [14] to develop an ABM that models the foraging and roosting patterns of American Robins and Eastern Bluebirds during the winter months (which we define as November 1 to March 1 [1]) in Northwest Indiana. To build our model, we made various simplifying assumptions; however, our results provide insight into the birds' general behavior in winter and

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this project provides a foundation for future research.

This paper is organized as follows: In Section 2 we give our agent-based model; Section 3 presents the results obtained from our simulations; we provide a discussion in Section 4; we conclude with ideas for future directions in Section 5.

## 2 Agent-Based Model

We outline the explanation of our agent-based model using the Overview, Design concepts, and Details (ODD) protocol recommended for the discussion of ABMs by Grimm et al. [3].

### 2.1 Purpose

The purpose of this agent-based model is to determine if an increased amount of food leads to a significant increase of winter survival rates of American Robins and Eastern Bluebirds in Northwest Indiana. To this end, our simulation considered various aspects known about thrush foraging behavior, thrush roosting behavior, and the environment of Northwest Indiana. The simulation results indicate the overarching characteristics of thrush behavior in this setting and how it changes with factors such as food availability and population density.

### 2.2 State Variables and Scales

This agent-based model is comprised of two primary components: bird agents and a landscape composed of various patches. Each bird agent represents either a robin or a bluebird. The landscape is a grid of patches representing the region of Northwest Indiana. These patches correspond to different habitat types with varying amounts of available food. These agents and patches correspond to those in the NetLogo program.

#### 2.2.1 Bird agents

Robins and bluebirds have comparable foraging, roosting, and migratory behavior, and so homogeneity was assumed among the two species in the model.

Each bird agent keeps track of its own energy level in kilocalories over the course of the simulation. At the beginning of the simulation, every bird agent has the same amount of energy at 300 kcal. This level is depleted over time according to the field metabolic rate (FMR) of American Robins. Additional energy is expended when the birds actively fly to a new location, while less energy is expended when they are sleeping. This energy level increases when the bird agent consumes fruit from the patch where it is located. When this energy level reaches

zero the agent is deleted and the bird is considered to have died.

Every bird agent also keeps track of the roosting site that it slept at the previous night. Birds' initial starting location is always a roosting site. The birds have a preference to return to the previous roosting site, however once food sources become depleted and birds have further to travel, other roosting sites also become attractive.

#### 2.2.2 Patches

The model's representation of Northwest Indiana consists of a  $31 \times 31$  grid of patches (for a total of 961 patches). The edges of this grid wrap so that the topology of the landscape is a torus. Each patch represents a 25 hectare ( $0.25 \text{ km}^2$ ) square area. This makes the entire model a  $15.5 \text{ km} \times 15.5 \text{ km}$  square region.

There are four different habitat types represented by patches in the model. Each habitat type contains a different concentration of food and is represented by a differently colored patch. Within each habitat type, patches are assigned an initial amount of food based on a normal distribution. Negative and non-integer values for the amount of fruit are not allowed. This fruit is depleted only when it is consumed by bird agents and there is no way for the amount of fruit in a patch to increase during the simulation.

Some patches in the modeled landscape serve as roosting sites to which bird agents must return to sleep for the night. Each patch has an equal probability of being a roosting site upon initialization. Roosting sites are represented by a darker colored patch. The number of roosting sites can easily be altered from one run of the model to the next using a slider bar on the NetLogo interface. However, we held the number of roosting sites constant at 15 in our large-scale simulations.

### 2.3 Process Overview

The model proceeds in timesteps of 30 minutes. For each timestep, every bird agent takes action to meet its needs according to a predetermined decision making process, which incorporates stochasticity as outlined in Figure 1.

#### 2.3.1 Bird Subroutines

The "Starve" subroutine checks to see if the energy level of the bird agent is less than or equal to zero. The bird agent is deleted and is said to have died if it has a non-positive energy value. If the bird agent has a positive energy value, then it continues in its decision-making process. This subroutine is invoked every time that a bird agent expends energy.

One of the first values that each bird checks in its decision making process is the time of day. If it is earlier

than 6:00 A.M., then the birds go about the “Sleep” subroutine. If it is between 6:00 A.M. and 10:00 P.M., then the birds go about their decision process when they are awake. After 10:00 P.M., the birds also go about the “Sleep” subroutine. These times of day were estimated and are not based on empirical field evidence of the birds’ sleep-wake cycle.

The “Sleep” subroutine first checks to see if the bird agent is in a roost. If the bird is in a roost, then bird simply expends the amount of energy calculated based on the bird’s BMR over a 30 minute period. If the bird is not in a roost, then the bird flies either to its previous roost or to the nearest roost based on a probability  $r_i$  comparing the bird’s distance from each roost in patch  $i$ . After arriving at a roost the bird sleeps for that timestep.

The “Move” subroutine is the first action that a bird agent takes every timestep that it is awake. This subroutine compares the patch where the bird is currently located to the surrounding patches visible to the bird agent. The bird then chooses the most attractive visible patch (highest  $p_i$ ) based on food quantity and distance from the bird’s current location. The bird then chooses whether or not to actually travel to this attractive patch based on a probability comparing it to its current patch based on these same values for each patch. In the case that all patches within the bird’s visible range have very little food, then the bird will choose a random direction to fly (taxi) in search of patches with more food. If a bird must taxi to a new location it travels a distance determined by a normal distribution with an average of 5 patches (2.5 km) and a standard deviation of 1 patch (0.5 km), rounded to the nearest non-negative integer. After the “Move” subroutine, the “Starve” subroutine is run because the birds have expended energy if they flew from one patch to another.

After a bird has decided how it will move, then it goes through the “Eat” subroutine. This subroutine determines how much food a bird agent will eat from its current patch and makes the bird eat it. The amount of food that a bird eats  $C$  is given in Equation (4). After the bird eats then its energy value is updated before the timestep then ends.

The patch agents do not change by themselves over time. They are only impacted by their interactions with the bird agents. As bird agents eat fruit, the amount of fruit in the patch is depleted accordingly.

## 2.4 Design Concepts

### Emergence

Some aspects of the model such as the bird’s survivability, the amount of fruit that they eat, and any tendencies to group together or disperse emerged from the behavior

of the individual birds and their interaction with the environment. Although the birds’ general decision making process is strictly defined, many of their actions and most of their choices incorporate stochasticity.

### Fitness

In the model, each bird tries to survive the winter as best it can by consuming as much fruit as it can, while limiting its energy expenditures from moving. This is accomplished primarily through the “Move” subroutine which determines if, where, and how birds move to meet this goal.

### Sensing

During a simulation bird agents are assumed to be aware of their environment within a radius of 5 patches (2.5 km). Within this radius, a bird agent is aware of the various patches along with their amount of food as well as the other birds in this area. A distance of 5 patches was chosen arbitrarily. This distance was decided on because a bird was assumed to be able to fly quickly and be roughly familiar with its territory—enough to be fully aware of its environment within a 5 patch radius. This distance was also sufficiently small to reasonably limit the number of patches that a bird agent will have to consider when deciding where to move.

### Stochasticity

Many aspects of the model incorporate stochastic processes. Within the environment, the locations of and thus the distribution of the roosts was completely random. Additionally, the initial amount of food in a given patch was determined by a normal distribution with an average determined by the type of patch. The type of each patch was also determined randomly so that each patch had an equal likelihood of being any particular type.

For the bird agents, the starting location of each bird was a randomly chosen roosting site. Many of the bird agents’ decisions involved stochastic processes. For instance, when a bird must fly to a roost for the night, it decides whether or not to fly to its previous roost or another one based on a probability related to the agent’s relative distance to the unfamiliar roost. Also, when bird agents eat fruit, the amount of fruit  $C$  is determined by random variable.

Finally, the “Move” method includes stochasticity in two ways. First, a bird agent will decide to move to the most attractive patch that it can sense based on a probability determined by comparing the prospective patch to the bird’s current patch. Second, in the event that a bird chooses to taxi because there are no observable patches with a certain amount of food, then the bird randomly

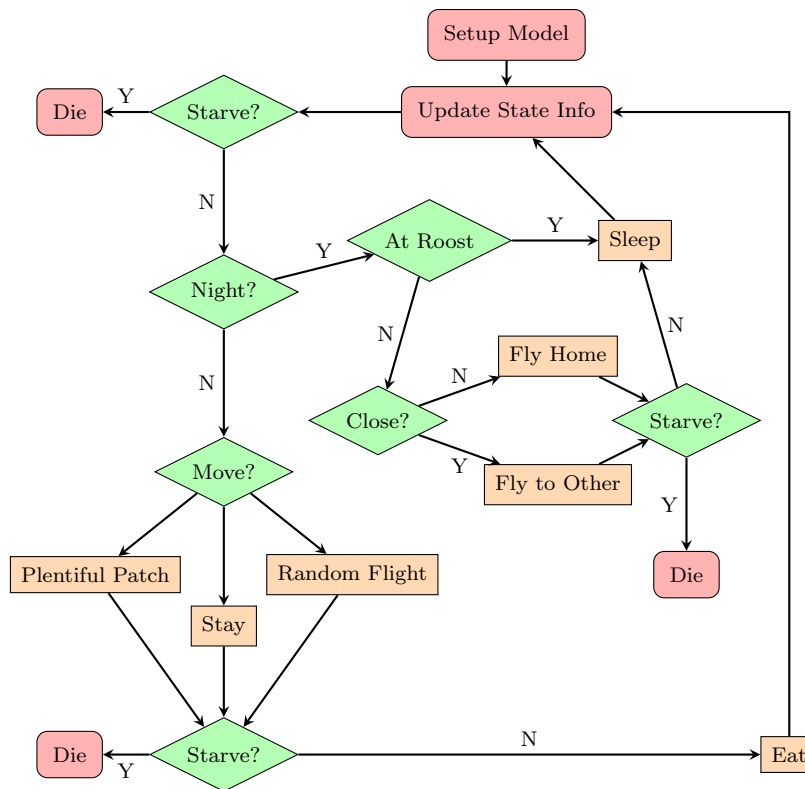


Figure 1: This decision tree describes a bird’s decision-making process for each timestep in the ABM.

chooses a direction to fly in and flies a distance determined by a normal distribution.

### 2.5 Initialization

At the beginning of each run of the simulation, the initial number of robins, bluebirds, and roosts is set by the observer using the slider bars on the Netlogo interface. Runs can be performed individually, by pressing “Go” on the Netlogo interface, or batch runs can be performed. The observer can select what data to output, and the Netlogo software outputs the desired information into an Excel spreadsheet. The data were then processed and evaluated in MATLAB.

### 2.6 Inputs

#### 2.6.1 Patches

There are four basic types of patches, each containing a different amount of food relative to the other types. Patches of type 1, 2, 3, and 4 have a “concentration”  $c$  of fruit of 0%, 30% , 60%, and 85%, respectively. The value of  $c$  is stored as a percent, so for type 2,  $c = 30$  as opposed to 0.3. The initial amount of fruit  $F_i$  in patch  $i$

is a random variable given by

$$F_i = \max([\text{round}(10XA)], 0), \tag{1}$$

where  $[\cdot]$  is the nearest integer function (specifically, we use NetLogo’s `round` routine, which rounds 1/2 up to 1),  $X \sim N(c, 100)$ , and  $A \in [0, 1]$  is the Food Abundance Percent. The Food Abundance Percent  $A$  is a variable that the user can adjust to scale the total amount of fruit in the simulation. For a bird agent, consuming one of these fruits increases its energy level by 3 kcal. Because of a lack of suitable literature on the total amounts of fruit or on the energy contained in the different types of available fruits, these values were estimated. However, since the total amount of food was one of the values that we varied in our simulations by modifying  $A$ , the results for various values were considered in that way.

#### 2.6.2 Energy Expenditure

The basal metabolic rate we calculated for the birds was

$$\frac{0.003644256 \text{ kcal}}{\text{grams of bird} \cdot 30 \text{ min}}. \tag{2}$$

We calculated this using thermodynamic values and stoichiometry found in a general chemistry textbook [12].

This value was used to determine the bird’s energy expenditure while sleeping.

The field metabolic rate we calculated for the bird agents is 1.11536 kcal per 30 min timestep. This value was calculated from the regression equation relating FMR to mass in passerines found by Kenneth Nagy in his study on FMR in different types of animals based on mass [9]. The average weight of the robins was used to calculate the FMR for the bird agents. This gave us a value of approximately 225 kJ per day, which was then converted to 1.11536 kcal per half-hour. This was the value that was used as an energy cost for the bird agents per timestep while they were awake.

The mass of the birds was calculated using data from two studies that included a variety of birds, one of which was the American Robin. A study performed by Christopher Guglielmo had the average body mass of 18 American Robins from Ontario, Canada [4]. Another study performed by Alexander Gerson had a mean pre-flight mass for 6 robins [2]. We used these body mass values and the given standard deviations to obtain an average body mass for our study. We obtained an average of 77.0808 g, which was the mass utilized in the Netlogo model. This mass was used for both American Robins and Eastern Bluebirds.

The energy cost for the flight of the bird agents was 8.167 kcal per half kilometer (patch) flown. The study performed by Alexander Gerson provided values for the energy content of lean mass and fat mass used for robins. We also used Gerson’s experimentally determined average values for the amount of lean mass and fat mass consumed per minute of flight [2]. These values were used to calculate an energy expenditure per minute of flight. Then a reasonable flight speed for the American Robin was obtained from the book *The American Robin* by Roland Wauer. We took the middle of the range of flight speeds specified by Wauer, giving us a flight speed of 24.5 mph for robins [13]. This speed was used to convert our energy cost per minute to energy cost per half kilometer (patch) flown. Our final value of 8.167 kcal per half kilometer (patch) flown was used as the energy cost for the bird agents’ movement between patches.

### 2.6.3 Equations

Within the “Move” subroutine, the bird agents need a way of ranking the surrounding patches based on their food density and distance away. A value for the bird’s attraction  $p_i$  to every observable patch  $i$  was determined by

$$p_i = \frac{25F_i/(F_0 + 0.001) - 0.8167d_i}{25F_i/(F_0 + 0.001) + 0.8167d_i + 0.001}, \quad (3)$$

where  $d_i$  represents the distance (in patches) from the bird agent to patch  $i$ ,  $F_i$  is the amount of fruit in patch  $i$ , and  $F_0$  is amount of fruit in the bird’s current patch. The patch  $i$  with the highest  $p_i$  value was the only destination considered. The value  $p_i$  was then used as the probability that the bird agent would fly to the new patch rather than stay at its current patch. The addition of 0.001 in denominators prevents division by zero. This method of evaluating the patches makes the bird agents balance both their ability to find fruit with their desire to not unnecessarily expend energy.

The equation determining how much fruit  $C$  a bird agent consumes in a particular timestep is given by a random variable based on the local fruit density given by

$$C = \max\left(\left[\frac{5F_0}{400 + F_0} + \mathcal{E}\right], 0\right), \quad (4)$$

where  $\mathcal{E} \sim N(0, 4)$ . This modified Holling Type II function was chosen because it reflects the property that even under very high food densities, there is a maximum amount of food that can be processed based on how quickly the agent can eat the food. Additionally, the equation yields lower quantities of fruit when the overall fruit density is lower. This aspect reflects the idea that a bird agent will be able to find less food when it is more scarce.

The bird agents must have a decision distribution for deciding what roost to go to. The probability  $r_i$  that a bird would choose to fly to a different roosting site than its previous one is

$$r_i = \max\left(\frac{d_{\text{home}} - d_i}{d_{\text{home}} + d_i}, 0\right), \quad (5)$$

where  $d_{\text{home}}$  is the distance from the bird to its previous roost and  $d_i$  is the distance from the bird to the potential new roost in patch  $i$ . This value was chosen because it causes the bird agent to be more likely to sleep at the unfamiliar roost the closer that it is to that roost relative to its previous roost. For instance, in the case that a bird agent is already in the same patch as the unfamiliar roost, then there is a 100% chance that it will simply stay there. However, this equation also causes the bird to always return to its previous roost if that happens to be closer. Although this relationship does not have an experimental basis, it was chosen for its reasonability and desirable properties within the model.

## 3 Results

We ran two large-scale simulation experiments. For each experiment, we looked at bird survivability in terms of another variable. The first experiment was run varying food availability, and its results can be seen in Figure 2.

The initial population in this experiment was 20 birds, with 15 available roosts. Survival increased with greater food availability, although not linearly.

The second experiment was run varying initial bird population and its results can be seen in Figure 3. In this experiment, food availability was 25, with 15 available roosts. Survival percentage decreased with a greater initial population.

Each data point is the average of  $n = 30$  realizations of the model. Standard deviations were small and, thus, not graphed.

The data from each experiment appeared to fit a sigmoid (logistic) curve, and thus, we fit all 30 realizations of each experiment simultaneously to the function

$$f(x) = \frac{L}{1 + \exp(-k(x - x_0))} \quad (6)$$

using MATLAB's `fminsearch` routine on an ordinary least squares cost functional. The parameter values for the best fitting curves as well as the  $r^2$  values are given in the captions of the respective figures.

## 4 Discussion

This model was constructed to investigate the question of if American Robins and Eastern Bluebirds are overwintering in Northwest Indiana due to an increased winter food supply from invasive plants. Our results confirm our biological intuition. In the first simulation experiment we ran, we varied the amount of food that was available for consumption. As the food abundance increased, the number of birds able to survive also increased. Since the initial number of birds in the simulation is a fixed value, one would expect that with more food available, more birds will be able to survive due to a decrease in competition between birds. The results from this simulation verify that with more food available, more birds survive.

In the second simulation experiment we varied the number of birds and held the amount of food constant. The results tended to verify the biological intuition that with competition, less birds survived when the initial bird population was high rather than when the bird population was low. This also follows from the reality of competition for resources. If there were large amounts of birds foraging for some amount of food, it would be less probable that a bird would survive compared to if there were only a few birds foraging for the same amount of food.

As the results of the first simulation experiment indicate, more resources leads to higher winter survival of birds. It is certainly possible that the presence of invasive plants are causing American Robins and Eastern Bluebirds to winter in Northwest Indiana and not migrate south. However, a number of important factors should

still be considered for the model and will be outlined in the following section.

## 5 Future Work

The research performed for this project has many potential areas for future development and investigation. While the model we have constructed acts as a foundation, there are many biological factors that should be included. The model is set up in such a way that the bird populations and food resources can be altered according to the biologically empirical values, however these numbers are yet to be determined.

Since the goal of this project is to determine if thrushes are wintering in Northwest Indiana and choosing not to migrate, it would be necessary to distinguish birds that migrated into the area from the birds that remained in the area for the winter season. While this would be simple to track using an ABM, being able to compare this to field data would be tricky as the latter would be difficult to obtain.

Some simplifying assumptions were made due to time restrictions on our research, however further research could include collecting biological data to input where simplifying assumptions were made when creating the model. Currently, as stated in Section 2.2.1, the only difference between the American Robin and Eastern Bluebird is the color of the agent in the Netlogo interface. While the birds do have similar roosting and migration patterns, they surely have differences that would become apparent with further investigation. These differences could then be included in the model to create a more accurate simulation of their respective behavior.

We also made the simplifying assumption that the only way the food resource decreased was via bird interaction. This is not an accurate representation, as there are natural causes that contribute to food depletion, one of which includes competition for resources within the environment. With further biological investigation, a more realistic representation of biological competition for food resources could be included in the model.

Natural causes of food depletion could also be included. For example, Northwest Indiana is known to experience harsh winters, oftentimes with blizzards and lake effect winter conditions, which could affect the food availability over the course of the winter. Likewise, these harsh conditions could cause death of the birds themselves, and such factors are not included in the current model. Further research could be conducted to include data of meteorological trends and biological consequences regarding the realistic ability a bird has to survive during such conditions.

In the same way, these conditions most likely affect the

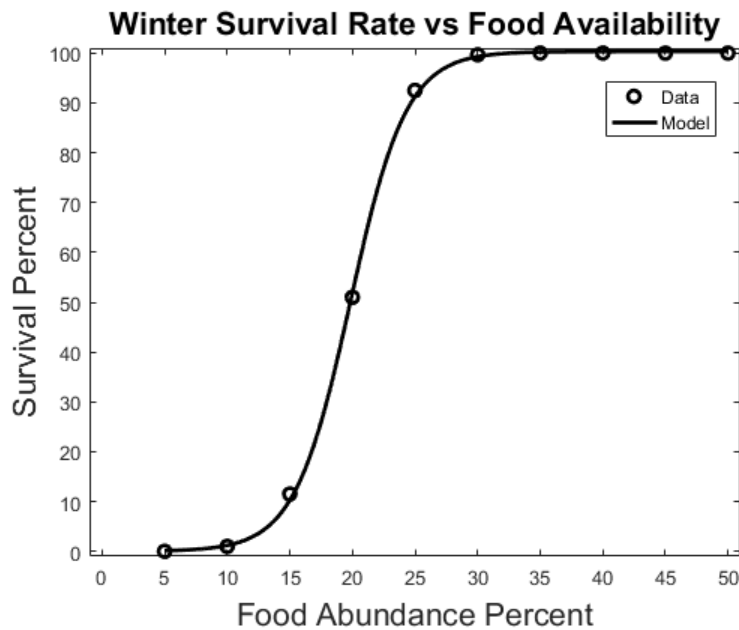


Figure 2: Number of birds alive at the end of the winter months versus food availability. Each point on the “Data” (circles) curve is the average of  $n = 30$  realizations of the ABM. The “Model” (solid) curve is the best fitting (to the full data set) logistic curve given by Equation (6), with parameter values  $L = 100.3281$ ,  $k = 0.4482$ , and  $x_0 = 19.8481$ , along with an  $r^2 = 0.9632$ .

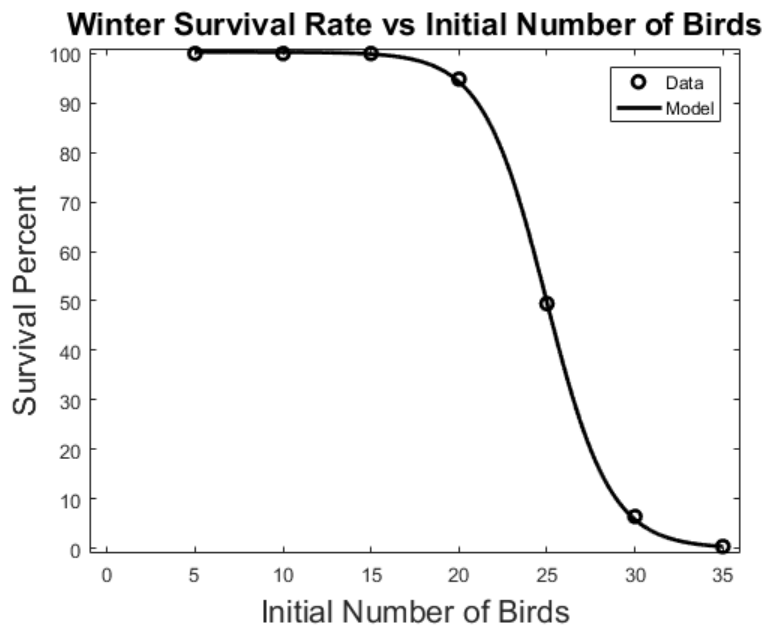


Figure 3: Percentage of birds alive at the end of the winter months versus initial population. Each point on the “Data” (circles) curve is the average of  $n = 30$  realizations of the ABM. The “Model” (solid) curve is the best fitting (to the full data set) logistic curve given by Equation (6) with parameter values  $L = 100.2901$ ,  $k = -0.5533$ , and  $x_0 = 24.9700$ , along with an  $r^2 = 0.9598$ .



foraging and roosting behavior of the birds. Since a meteorological homogeneity was assumed, further investigation could provide a clearer and more accurate depiction of the effect the severe weather conditions have on birds' foraging patterns as well as their energy expenditure and roosting patterns.

With the inclusion of these additional factors, we believe it would be possible for experiments to be performed with the ABM that could conclude whether or not thrushes are wintering in Northwest Indiana specifically due to the increased resources offered by the invasive plants.

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