ANALYSIS OF STEADY STATES FOR CLASSES OF REACTION-DIFFUSION EQUATIONS WITH U-SHAPED DENSITY DEPENDENT DISPERSAL ON THE BOUNDARY

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ABSTRACT. We consider positive solutions to equations of the form

$$\begin{cases} -\Delta u = \lambda u(1-u), & x \in \Omega, \\ \frac{\partial u}{\partial \eta} + \gamma\sqrt{\lambda}(u-A)^2 u = 0, & x \in \partial\Omega, \end{cases}$$

where $\lambda > 0, \gamma > 0, A \in (0,1)$ are parameters, Ω is a bounded domain in $\mathbb{R}^n$; $n \geq 1$ with smooth boundary $\partial\Omega$ and $\frac{\partial u}{\partial \eta}$ is the outward normal derivative. Such models arise in the study of population dynamics in a habitat Ω when the population exhibits U-shaped density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness and multiplicity of positive solutions) as the patch size (λ) and the hostility of the outside matrix (γ) vary. We obtain results when $\Omega = (0,1)$ via a quadrature method, and when Ω is any bounded domain in $\mathbb{R}^n$; $n > 1$ by the method of sub-super solutions.

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