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Megan Eberle
University of Wisconsin-La Crosse, eberle.megan@uwlax.edu

Eric A. Eager
University of Wisconsin-La Crosse, eeager@uwlax.edu

James Peirce
University of Wisconsin-La Crosse, jpeirce@uwlax.edu

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How Infectious Was #Deflategate?

Megan Eberle[1], Eric Eager[1,*], James Peirce[1]

*Correspondence:
Prof. Eric Eager,
Dept. of Mathematics, UWL
1725 State St., La Crosse, WI
54601,
USA
eeager@uwlax.edu

Abstract
In mid-January 2015 the National Football League (NFL) started an investigation into whether the New England Patriots deliberately deflated the footballs used during their AFC Championship win. Like an infectious disease, the initial discussion regarding Deflategate grew rapidly on social media sites in the days after the release of the story, only to slowly dissipate as interest in the NFL waned following the completion of its season. We apply a simple epidemic model for the infectiousness of the Deflategate news story. We find that the infectiousness of Deflategate rivals that of many of the infectious diseases that we have seen historically and is actually more infectious than recent news stories of seemingly greater cultural importance.

Keywords: disease model, inverse problem, national football league, basic reproduction number

1 Introduction

The National Football League (NFL) has made American football arguably the most popular sport throughout the United States. The NFL was formed in 1922 from the American Professional Football Association and originally consisted of 18 teams. By 1925 the league began drawing tens of thousands of fans into stadiums to watch games live, and by 1934 the first NFL game between the Chicago Bears and the Detroit Lions on Thanksgiving Day was broadcasted live on national radio—allowing the league's fan base to spread nationally. As the radio and television became more widespread across the United States, the NFL was able to increase their popularity as almost all NFL games were broadcasted on radio or television by 1964, allowing professional football to overtake professional baseball in popularity around 1965. The popularity of the NFL continues to grow today as television ratings surge into the millions viewers and stadiums grow to house over 100,000 fans [13].

Despite the popularity of the NFL, its history has been plagued by numerous scandals, with the most recent scandal involving the footballs used by the New England Patriots in the 2015 AFC Championship game against the Indianapolis Colts, which has become known as "Deflategate". On Monday January 19, 2015, a story broke that the NFL had started an investigation into whether the New England Patriots deliberately deflated the footballs they used during their AFC Championship win over the Indianapolis Colts. The NFL rules state that footballs must weigh between 14 and 15 ounces and be inflated between 12.5 and 13.5 pounds per square inch. Deflated footballs would have given the Patriots a competitive advantage over the Colts (as well as other opponents) by allowing them to exploit the differences between an improperly-inflated football and a properly-inflated football [7].

Discussion regarding Deflategate was sparked instantly on social media and grew rapidly in the hours and days after the release of the story. The scandal gained national attention quickly as the New England Patriots had just earned a trip to Super Bowl XLIX against the Seattle Seahawks (another team displaying questionable ethics of its own in previous seasons [14]). Since the Super Bowl is the most anticipated game in the NFL season, attention to the story was heightened. After the Super Bowl was over, the scandal slowly began to dissipate and lost much of the attention it originally had, as interest in the NFL decreased at the completion of its season. This interest continued to dissipate until early May 2015, when the aftermath of a 243-page report by independent attorney Theodore V. Wells, Jr., resulted in the NFL suspending Patriots star quarterback Tom Brady for the first four games of the 2015 season and stripping the team of two high draft picks. In early September this suspension was nullified, allowing Brady to play the entire 2015 season [4].

The sharp rise in interest in the Deflategate scandal and then an initial slow dissipation of interest is similar to that of an outbreak of an infectious disease. When an infectious disease is first introduced to a new population, it can spread rapidly as a large number of the popula-

[1]Department of Mathematics, University of Wisconsin-La Crosse, La Crosse, WI

tion becomes infected with the disease. As more of the susceptible population becomes infected with the disease, the number of possible interactions between susceptible and infectious individuals, and thus the rate of new infections, decreases. The infectiousness of a disease can often be captured by a dimensionless parameter called the *basic reproduction number* $\mathcal{R}_0$. The value of $\mathcal{R}_0$ is the expected number of secondary cases produced by a single infected individual when introduced to a completely susceptible population. For example, the $\mathcal{R}_0$ value for the 2014 outbreak of Ebola virus in West Africa estimated between 1.51 and 2.53 [1], meaning that if one person was infected with Ebola during the early stages of the epidemic, they would likely spread the virus to approximately 2 other people before they recovered or died. Some other $\mathcal{R}_0$ values include 4 for the HIV and SARS, 10 for mumps and 18 for measles [12]. An $\mathcal{R}_0$ value of 18 is extremely high and means that measles is extremely infectious, which is why the disease is so catastrophic when it persists within a population.

Since the spread of the Deflategate scandal appears to be similar to an infectious disease, we compute the $\mathcal{R}_0$ value for this news event in order to quantify the story's infectiousness. To do this we must determine a useful medium through which information was spread, In this paper, we use the the social networking site Twitter (`http://twitter.com`). Twitter allows users post *tweets*, a message under 140 characters, to their Twitter page so that their followers, other users who will see the tweet, can read their message. The followers can then reply to the message and start a conversation with the owner of the tweet. They can also *retweet* that tweet to their followers. A retweet is when follower takes a tweet they saw and posts it onto their Twitter page for their followers to see. The number of tweets and retweets about Deflategate can be used to determine the $\mathcal{R}_0$ value and the infectiousness of this scandal as a news story.

In this paper we utilize a simple SIR epidemic model for the infectiousness of the NFL's Deflategate news story. We then use Twitter data to estimate the parameters of this model using standard techniques from the study of inverse problems. We find that the infectiousness (as measured by $\mathcal{R}_0$) of Deflategate rivals that of many infectious diseases and is actually more infectious than the story of Hillary Clinton's announcement of her presidential campaign in April 2015—both in terms of $\mathcal{R}_0$ and in terms of the average amount of time the average tweeter continued to tweet about the news story. We also show that the average individual tweeted about Deflategate more than ten times longer than they did about the story of Freddie Gray's death that elicited the Baltimore riots in April 2015 [16].

2 The Model

While the punishment of the Patriots in May 2015, and the subsequent lift of said punishment in September 2015, created their own news stories regarding Deflategate, we will consider only the dynamics of the initial story, occurring in January 2015. We assume that information concerning Deflategate is spread person to person like an acute infection. We will focus on the subpopulation of media consumers that use the social media site Twitter. Time t will be measured in days since the first report of Deflategate on January 19th, 2015. Using the terminology of Kermack and McKendrick [20] we categorize the individuals in the Twitter population as susceptible, infectious, and removed. The *susceptible population* $S(t)$ (measured in thousands) are those individuals that regularly read and comment on sports news but have yet to comment on Deflategate as of time t. The *infectious population* $I(t)$ (measured in thousands) are those that are tweeting (or retweeting) posts about Deflategate using the keywords #deflategate, deflate gate, deflate-gate, spygate, or "deflated balls" (chosen to match available data). As a simplifying assumption we consider each tweet as representing a unique individual (tweeter), and thus tweets and individuals in the infectious class are discussed interchangeably. The *removed population* $R(t)$ (measured in thousands) includes either those individuals who do not read or comment on sports (and are hence removed from this news event even from the very beginning), or individuals that were once tweeting about the story and have permanently stopped using the Deflategate keywords. Due to the short lifespan of the story (on the order of weeks), we assume that the immigration of new users and emigration of current users can be neglected, yielding a constant total population size.

The progression from the susceptible population S to the infectious population I to the recovered (or maybe more appropriately named bored) population R can be visualized in the traditional SIR conceptual diagram (Figure 1). The progression of the information through the population depends on many factors. One of the most prominent is the total number of "interactions" between susceptible and infectious individuals. Information about Deflategate spreads when a susceptible individual comes in contact with the information spread by an infectious individual (by reading his or her tweets) and subsequently becomes infectious (starts tweeting themselves). Mathematically, a reasonable measure for the number of encounters between susceptible and an infectious individuals, assuming homogeneous mixing, is given by the product SI. This is referred to as the *law of mass action* in the applied mathematics literature [18]. However, not every interaction of a susceptible person reading a tweet about Deflategate will cause a retweet or a series of origi-

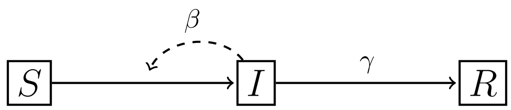

Figure 1: Conceptual model of information transmission and recovery into the removed population. Black arrows show the movement between the S and I classes and the I and R classes. The fact that the size of the infected population influences the rate at which a susceptible individual moves in to the infectious class is shown by the dashed arrow.

nal tweets by the newly-infectious. We use a parameter β, the transmission coefficient, to represent the probability a susceptible reader will retweet, per day, per thousand infected individuals. Once infectious, we assume that individuals tweet about this story for an average of $1/\gamma$ days. While it may be possible, especially for long-living news stories, that individuals can go directly from susceptible to recovered/bored, we assume that if an individual starts in the S class they either stay there for the duration of the story or pass through I before entering R. These assumptions can be collected to create the following SIR model

$$\begin{aligned} S'(t) &= -\beta S(t)I(t), \\ I'(t) &= \beta S(t)I(t) - \gamma I(t), \\ S(0) &= S_0, \\ I(0) &= I_0, \end{aligned} \tag{1}$$

where S_0 and I_0 are parameters giving the initial population of susceptible and infectious individuals (measured in thousands). Since the $R(t)$ population doesn't interact with the rest of the population at any given time, we can omit it from our analysis. We assume that the parameters β, γ, S_0 and I_0 are all positive, which implies that the solution vector $[S(t) \;\; I(t)]^T$ exists and is positive for all $t > 0$ [23].

The derivation of the basic reproduction number $\mathcal{R}_0$ for this SIR model follows from determining the stability threshold for the "disease-free" state $[S_0 \;\; 0]^T$, where the susceptible population has yet to be exposed to the news story. As with classical disease models (see [2, 19] for examples), the basic reproduction number is computed to be $\mathcal{R}_0 = \beta S_0/\gamma$. One can interpret this value as the total rate of initial infections (βS_0) times the average amount of time spent infected (γ^{-1}). Thus, for fixed S_0, diseases with large β and/or small γ will be the the most likely diseases to have $\mathcal{R}_0 > 1$, which results in an epidemic.

3 Methods

3.1 The #Deflategate Model

The size of the infected population during the first days of the Deflategate story was gleaned from an article [9] published in Boston Magazine. Data presented in the article was compiled by the website Topsy (http://www.topsy.com), an analytics company and certified Twitter partner, that collects Twitter data over the span of 30 days. The article limited the Topsy data to the keywords #deflategate, "deflate gate", "deflate-gate", "spygate", or "deflated balls". The number of tweets per day using those keywords is given by the unfilled circles in Figure 2.

We used this Twitter data to reverse-engineer the values of the parameters β, γ, S_0, and I_0 in the model (1) by using standard methods from the study of inverse problems [6], which we summarize below.

3.1.1 Parameter Estimation

To employ the techniques from inverse problems we require a *statistical model* to go along with the mathematical model (1). To create such a statistical model, we abstract our mathematical model (1) as in [5] to give us

$$\begin{aligned} \vec{x}'(t, \vec{\theta}) &= \vec{g}(\vec{x}(t, \vec{\theta}), \vec{\theta}), \quad t \in [t_0, t_f], \\ \vec{x}(t_0, \vec{\theta}) &= \vec{x}_0, \end{aligned} \tag{2}$$

where $\vec{x}(t, \vec{\theta}) = [S(t, \vec{\theta}), \; I(t, \vec{\theta})]^T$ and $\vec{x}_0 = [S_0, \; I_0]^T$ are the vectors of state variables and initial conditions of our system, given the parameter vector $\vec{\theta} = [\beta, \; \gamma, \; S_0, \; I_0]^T$. We define as our *observation process* the following

$$y(t) = f(t, \vec{\theta}) = C\vec{x}(t, \vec{\theta}) = I(t, \vec{\theta}),$$

as we are only aware of the infectious population (i.e., the tweeters) at any given time t. In this case $C = [0 \; 1]$ is a functional over $\mathbb{R}^2$. If we were able to see the entire system $[S(t, \vec{\theta}) \;\; I(t, \vec{\theta})]^T$ at any given time, then the operator C would be the 2×2 identity matrix.

To find an estimate for the parameter vector $\vec{\theta}$ we formulate the statistical model

$$\begin{aligned} Y(t) &= f(t, \vec{\theta}_0) + \epsilon(t) \\ &= I(t, \vec{\theta}_0) + \epsilon(t), \quad t \in [t_0, t_f], \end{aligned}$$

where $\vec{\theta}_0 = [\beta_0, \gamma_0, \; (S_0)_0, \; (I_0)_0]$ is considered the vector of hypothesized "true values" of the unknown parameters and $\epsilon(t)$ is a random variable that represents observation error for the observed state variable at each time t. We assume that the error function ϵ is such that $E(\epsilon(t)) = 0$,

$Var(\epsilon(t)) = \sigma^2$, and $Cov(\epsilon(t)\epsilon(s)) = \sigma^2\delta(t-s)$ for all $t, s \in [t_0, t_f]$. Here $\delta(t-s)$ is the Dirac delta function centered at s and σ^2, the assumed variance between the data and the model, is assumed constant but not necessarily known.

Since our data is collected at discrete time points, we have to write our above statistical model in discrete terms. Given data $I_1, I_2, \ldots, I_n$ taken at time points $t_0 \le t_1 < t_2 < t_3 < \cdots < t_n \le t_f$ we write our observation process as

$$f(t_j, \vec{\theta}) = I(t_j, \vec{\theta}), \;\; j = 1, 2, \ldots, n,$$

with the discrete statistical model as

$$Y_j = f(t_j, \vec{\theta}_0) + \epsilon(t_j) = I(t_j, \vec{\theta}_0) + \epsilon(t_j).$$

Knowing the value $\vec{\theta}$ and solving the system of differential equations (1) with these parameter values is known as the *forward problem*. Alternatively, having a set of data $I_1, I_2, \ldots, I_n$ and estimating $\vec{\theta}_0$ is known as solving an *inverse problem*. In this paper we perform the latter.

While several methods exist to solve inverse problems [5], we will use ordinary least squares. In addition to the assumptions above, we assume that realizations of $\epsilon(t)$ at particular time points are independent and identically distributed normal random variables. In this case, one can show that the parameter vector $\vec{\theta}_0$ that maximizes the likelihood function

$$\mathcal{L}(\vec{\theta}_0) = P(I_j = I(t_j, \vec{\theta}), j = 1, 2, \ldots, n | \vec{\theta} = \vec{\theta}_0)$$

is given by the parameter vector that minimizes the least-squares functional

$$\mathcal{LL}(\vec{\theta}_0) = \sum_{j=1}^{n} (I_j - I(t_j, \vec{\theta}))^2.$$

In other words

$$\vec{\theta}_0 = \text{argmin}_{\vec{\theta} \in \Theta} \mathcal{LL}(\vec{\theta}), \tag{3}$$

where Θ is the set of admissible values for the parameter vector $\vec{\theta}$. For the model in this paper

$$\Theta = (0, \infty) \times (0, \infty) \times (0, 3.02 \times 10^5) \times (0, 3.02 \times 10^5),$$

where 3.02×10^5 are the estimated number of monthly Twitter users (in thousands) as of 5/14/2015 [26].

All programming was performed in R [22], with code available upon request. We used fourth-order Runge-Kutta [25] to solve the forward problem (2) for each set of parameters $\vec{\theta}$ in Θ and the built-in optimization algorithm "optim" in R [22] to solve the minimization problem (3) for $\vec{\theta}_0$, the solution of the inverse problem.

3.1.2 Sensitivity Analysis

Once an estimate for parameter vector $\vec{\theta}_0$ is found, the next question is in regards to the uncertainty in the estimate. To generate standard errors for each of the parameters we create a 1×4 *sensitivity matrix*

$$D_j(\vec{\theta}) = \left[\frac{\partial I(t_j, \vec{\theta})}{\partial \theta_1} \;\; \frac{\partial I(t_j, \vec{\theta})}{\partial \theta_2} \;\; \frac{\partial I(t_j, \vec{\theta})}{\partial \theta_2} \;\; \frac{\partial I(t_j, \vec{\theta})}{\partial \theta_2} \right],$$

from which we create the 4×4 covariance matrix

$$\Sigma^n(\vec{\theta}_0) = (\sigma^2)^{-1} \left(\sum_{j=1}^{n} D_j^T(\vec{\theta}_0) D_j(\vec{\theta}_0) \right)^{-1}.$$

It follows that the standard error in the kth component of the parameter vector $\vec{\theta}_0$ can be approximated by the square root of the (k,k)th element of the matrix $\Sigma^n(\vec{\theta}_0)$ [5]. In the Results section, we report parameter values in an interval with the lower and upper bounds being two standard errors below and above the estimated value, respectively.

3.2 Comparison Stories

To put the Deflategate analysis into perspective, we solve an inverse problem for two other prominent news stories that happened near the time of the Deflategate story: the announcement of Hillary Clinton's presidential campaign and the stories surrounding the Baltimore riots resulting from the death of Freddie Gray. Interestingly, the SIR model used in this paper was not able to capture the behavior of either story, due to the almost immediate emergence of individuals tweeting about the story (see Figure 3 and Figure 4). Instead of using the traditional SIR model in these cases we used an SIR model allowing for recruitment of new tweeters for the Clinton story,

$$\begin{aligned} S'(t) &= \Lambda - \beta S(t) I(t), \\ I'(t) &= \beta S(t) I(t) - \gamma I(t), \\ S(0) &= S_0, \\ I(0) &= I_0, \end{aligned} \tag{4}$$

where Λ is rate at which susceptible tweeters increases through recruitment per day. In the Freddie Gray case, we used a simple exponential model,

$$I'(t) = -\gamma I(t)$$

to track the decay of tweets after the emergence of the story. In the former model we are still able to recover the basic reproduction number $\mathcal{R}_0$ and the average time $1/\gamma$ spent tweeting about the story. In the latter model we are only able to obtain an upper bound for $1/\gamma$.

4 Results

From the data in Figure 2, a parameter vector that minimized the value of $\mathcal{LL}$ is given by

$$\vec{\theta}_0 = [\beta_0 \;\; \gamma_0 \;\; (S_0)_0 \;\; (I_0)_0]^T = [0.010 \;\; 0.243 \;\; 157 \;\; 2.28]^T.$$

It's important to note that since $S(t)$ and $I(t)$ are in thousands of tweeters, so are $(S_0)_0$ and $(I_0)_0$. Both the initial $S(t)$ and $I(t)$ components of the solution to (1) subject to these parameter values are displayed in Table 1, along with the Twitter data used to solve the inverse problem. In addition, we include 95-percent confidence intervals for the parameters (Table 1). Since all of these confidence intervals exclude zero, we can be fairly certain that each of the parameters in the model are significantly different than zero, and thus necessary to include. While there were other parameter vectors that produced (local) minimums of $\mathcal{LL}$ in the space Θ, the solutions elicited by said parameter vector did not fit the data as well as the parameter vector above, producing larger values of $\mathcal{LL}$ and graphs less convincing than Figure 2.

The parameter values in $\vec{\theta}_0$ are able to give us quantitative information regarding the infectiousness of the Deflategate news story. For example, the estimate for β suggests that between 0.9 and 1.2 percent of susceptible tweeters' views of Deflategate tweets per day will result in the immediate conversion of said tweeter into someone tweeting about the Deflategate story per day. The estimate for γ suggests that the average Deflategate tweeter spends between 3.37 and 5.29 days tweeting about the Deflategate story before becoming bored with the story. Our estimates in $\vec{\theta}_0$ suggest a basic reproduction number between 4.21 and 11.05 new Deflategate tweeters caused by the initial Deflategate tweeter, a number rivaling some of the aforementioned epidemics in history.

When viewing S and I together on the same set of axes we see that by the sixth day of the news story the number of individuals has passed its maximum value. At this point both the S and I populations are decreasing as the story likely moves out of the news headlines. On the other hand, the values of I continue to be well above zero almost two weeks into the story. These two pieces of information suggest that this news story—and possibly NFL news stories in general—are quite infectious and have a relatively high amount of staying power, which coincides with our initial intuition [13].

5 Discussion

In this paper we applied, analyzed and fit a model for the infectiousness of the NFL's initial Deflategate story using a standard SIR epidemiological model and data from the popular social networking site Twitter. We found that the standard SIR model fit the data quite well (see Figure 2), suggesting that the assumptions inherent in using such a model are reasonable. In fact, there have been some studies (for example, [21] and [8]) suggesting that mass-action assumptions may be improperly applied in the study of actual epidemics when the underlying populations are (1) too small, (2) not homogeneous in space enough to warrant such a simple transmission probability, or (3) too crowded so as to saturate infectiousness when pushed beyond a certain population size. Populations of Twitter users, however, consist of many individuals on one webpage unimpeded by physical limitations, alleviating the aforementioned issues. Thus, the simplest SIR model may, in many ways, be a better initial model for some instances of information moving through a social networking site like Twitter than it is for an epidemic moving through a real population.

We used the parameters from the study of this inverse problem to determine how popular and persistent this news story was in terms of the composite parameter $\mathcal{R}_0$ and the parameter γ, respectively. We found that the average individual tweeting about the Deflategate story was able to elicit 4.21 to 11.05 new tweeters to tweet about Deflategate during the early stages of the news story, and that the average individual tweeting about the story tweeted between 3.37 and 5.29 days about the story.

We found that Hillary Clinton's announcement of her presidential bid, while having more total tweets at its peak than the Deflategate story (see Figure 3), and a similar initial infectiousness $\mathcal{R}_0 \in (3.38, 4.69)$, had far less staying power. The average tweeter was only tweeting about Mrs. Clinton's announcement for between 0.585 and 0.646 days. This may be due to the fact that the election was still more than a year away, or that there were other candidates announcing their bids during the same period of time. When studying the Baltimore riot story we found the sociological results of the parameter estimation to be similar. This story, while eliciting far more tweets than the Deflategate story, saw the average person tweeting about the story doing so for a time whose upper bound is between only 0.3227 and 0.3232 days (see Figure 4). One explanation for this short staying time would be that, after many instances of violent police actions over the course of the past year, many people are starting to grow fatigued by such stories.

The results in this manuscript suggest that the NFL's popularity rivals (even surpasses) that of what many find to be some of our nation's biggest news stories. Observations made in the wake of various influential news stories involving the intersection of a football with some of our nation's most contentious issues (gambling, equality, domestic abuse, child abuse and financial literacy, to name a few) appear to coincide with these results ([15], [3], [11],

Parameter	lower bound	estimate	upper bound
β	0.009	0.010	0.012
γ	0.189	0.243	0.297
S_0	139	157	174
I_0	0.665	2.28	3.89

Table 1: Parameter values and their respective 95% confidence intervals.

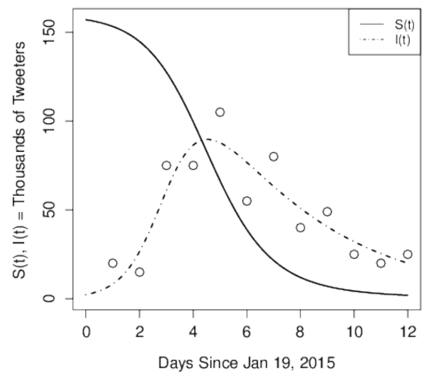

Figure 2: The number (in thousands) of Twitter users susceptible, $S(t)$, and tweeting about Deflategate, $I(t)$, subject to parameter vector $\vec{\theta}$ solving the inverse problem using Topsy.com data (unfilled circles). Here, $t = 0$ corresponds to January 19, 2015.

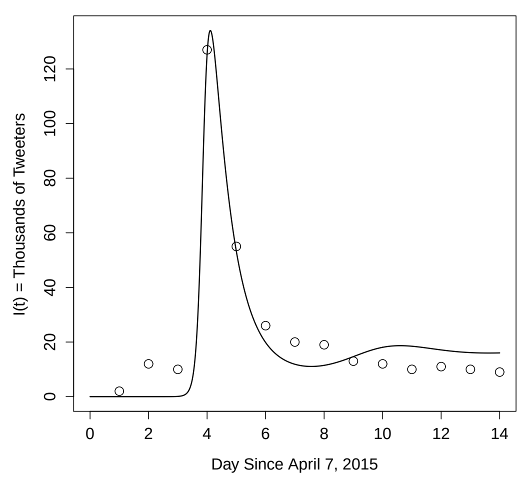

Figure 3: The number (in thousands) of Twitter users tweeting about Hillary Clinton's announcement for candidacy for the 2016 presidential race subject to the parameter vector $\vec{\theta}$ solving the inverse problem using Topsy.com data (unfilled circles). Here, $t = 0$ corresponds to April 7, 2015.

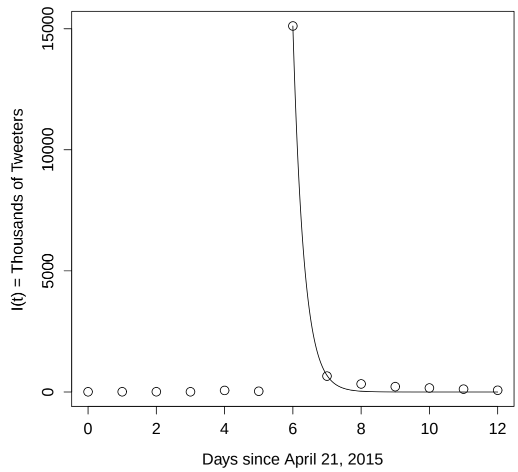

Figure 4: The number (in thousands) of Twitter users tweeting about Freddie Gray and the Baltimore Riots subject to parameter vector $\vec{\theta}$ solving the inverse problem using Topsy.com data (unfilled circles). Only the second half of the data (starting with $t = 6$) was fit to a simple exponential model due to the inability to fit an SIR model to all of the data. Here, $t = 0$ corresponds to April 21, 2015.

[24], [17], [10]). However, future work using mathematical modeling of infectious diseases applied to social networks requires a robust classification of various news stories so as to properly assess society's relative appetite for different types of stories and how this appetite is evolving in time.

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