2024

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Recommended Citation

Deal, E., Mooney, E., Cullen, A., Kroesch, A., Priya Bajwa, N., Corven, J., & MacDonald, B. (2024). Accessing and Assessing Components of Elementary and Middle School Students' Mathematical Disposition Through Metaphors. Investigations in Mathematics Learning, 1–17. https://doi.org/10.1080/19477503.2024.2419291

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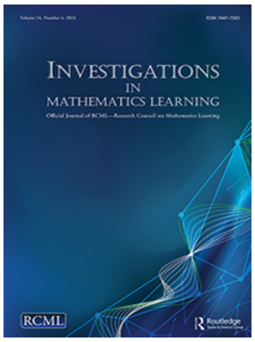

Investigations in Mathematics Learning

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/uiml20

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To cite this article: Emily Deal, Edward Mooney, Amanda Cullen, Allison Kroesch, Neet Priya Bajwa, Julien Corven & Beth MacDonald (11 Nov 2024): Accessing and Assessing Components of Elementary and Middle School Students' Mathematical Disposition Through Metaphors, Investigations in Mathematics Learning, DOI: 10.1080/19477503.2024.2419291

To link to this article: https://doi.org/10.1080/19477503.2024.2419291

Published online: 11 Nov 2024.

INVESTIGATIONS IN MATHEMATICS LEARNING
https://doi.org/10.1080/19477503.2024.2419291

Accessing and Assessing Components of Elementary and Middle School Students' Mathematical Disposition Through Metaphors

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ABSTRACT

This study examined student-generated metaphors comparing food to math as a means of accessing and assessing components of prekindergarten through Grade 8 students' ($N = 306$) mathematical disposition. Previous research provided insight into predominantly affective components of older students' mathematical disposition (i.e. Grade 4 through college). However, we extend these findings to include nonaffective components and younger students. Our study highlights the value in conceptualizing mathematical disposition as the sum of three mental functions, namely cognitive (e.g., mental processes such as reasoning), affective (e.g., attitudes, feelings, beliefs about mathematics or oneself as a learner), and conative (e.g., effort, grit, or level of challenge), and whose components may span multiple categories of these mental functions. Our study also extends the research on mathematical disposition by revealing that it is conditional in addition to being complex. Furthermore, several notable grade-based trends from data emerged in relation to students' enjoyment of mathematics as well as their views regarding the applicability, prevalence, and variety and complexity of mathematics.

KEYWORDS

Elementary; mathematical disposition; mental functions; metaphor; middle school

Many of today's educators and researchers define the learning of mathematics as more than the mere acquisition of content-related facts and procedures (e.g., Gresalfi & Cobb, 2006, National Council of Teachers of Mathematics, 2020). Because "what is learned cannot be separated from how it is learned" and experienced (cf. Beach, 1999; Boaler, 1997; Cobb & Bowers, 1999; Gresalfi & Cobb, 2006, p. 50; Lave, 1988), students' mathematical dispositions are receiving considerable attention (e.g., Beyers, 2011; Graven, 2012; Kamid et al., 2021; Young et al., 2021). Research indicates a positive disposition toward mathematics may increase the likelihood that students will seize opportunities to learn mathematics (Kilpatrick et al., 2001), increase motivation for learning (Cobb & Hodge, 2002), help students engage with mathematics (Ames & Archer, 1988; Gresalfi & Cobb, 2006), and positively impact students' overall success and achievement in mathematics (Kilpatrick et al., 2001; NCTM, 1989, 2000). In contrast, a negative disposition toward mathematics is said to limit students' opportunities to learn mathematics (Beyers, 2011) and cause students to avoid (or give up on) engaging in challenging problems (Beyers, 2011) or enrolling in advanced mathematics courses altogether (Cai et al., 2012). The field has traditionally recognized the influence of mathematical disposition on learning as either positive or negative, and as such has not yet fully uncovered how to productively and completely access and assess multiple components of students' mathematical disposition for all

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grade bands (Graven, 2012). As such, the focus of this study was to examine more closely components of mathematical disposition in prekindergarten (PK) through Grade 8 students.

Conceptual Framework for Mathematical Disposition

Although mathematical disposition has been conceptualized in a variety of ways, consistent among these conceptualizations is the recognition that disposition reflects a tendency, habit, or inclination to think or act in certain identifiable ways (Kilpatrick et al., 2001; McIntosh, 1997; NCTM, 1989). In this study, we adopted Beyers (2011) conceptualization of mathematical disposition as the "cognitive, affective, and conative functions that a student of mathematics tends or is inclined to engage or espouse in a mathematical context" (p. 72). In other words, we see mathematical disposition as the sum of cognitive (e.g., mental processes such as reasoning), affective (e.g., attitudes, feelings, beliefs about mathematics or oneself as a learner), and conative (e.g., effort, grit, or level of challenge) components that can be accessed and assessed.

The affective category of mental functioning refers to the feelings, emotions, moods, and temperament students experience with respect to mathematics (Beyers, 2011). It also encompasses beliefs about oneself as a learner of mathematics (Beyers, 2011; McLeod, 1992), attitudes toward mathematics (Beyers, 2011; McIntosh, 1997; McLeod, 1992), beliefs about the nature of mathematics (Beyers, 2011; Fernandez & Cannon, 2005; McLeod, 1992), beliefs about the usefulness of mathematics, beliefs about whether learning mathematics is worthwhile, and beliefs about whether mathematics is sensible (Beyers, 2011; Kilpatrick et al., 2001). The cognitive category of mental functioning refers to processes of perception, recognition, conception, judgment, and reasoning (English & English, 1958). It also pertains to making mathematical arguments or connections (Beyers, 2011; McClain & Cobb, 2001). And lastly, the conative category of mental functioning pertains to effort, diligence, and persistence (Beyers, 2011). By operationalizing mathematical disposition with affective, cognitive, and conative constructs, we are given a multi-faceted lens with which to examine the complexity of several relationships.

Literature on Accessing and Assessing Students' Mathematical Disposition

Researchers have used a variety of tools and approaches to access and assess students' mathematical disposition. The following section outlines a few of the field's previous attempts.

Likert-Scale Items

Large international and national studies have incorporated surveys to access components of students' mathematical disposition. The National Center for Education Statistics' (NCES) Trends in International Mathematics and Science Study (TIMSS) used four-point Likert scale items to access and assess students' enjoyment of mathematics (e.g., NCES, 2019). Similarly, the National Assessment of Educational Progress' (NAEP) Mathematics Student Questionnaire (e.g., NAEP, 2022), utilized five-point Likert scale items to access and assess components related to students' enjoyment of mathematics. While both surveys provided insight into mathematical disposition, they were narrow and limited due to the fixed nature of responses set by researchers. Furthermore, these surveys were single-dimensional. These limitations are notable as components of mathematical disposition are multidimensional (Kanak Pervez et al., 2020).

Furthermore, the polarization of response choices provided on the TIMSS (e.g., NCES, 2019) assessment did not provide students with a neutral option. This limitation forced a positive or negative expression of attitude on respondents (Lavrakas, 2008). This is consistent with literature's current categorization of mathematical disposition as either positive or negative (e.g., Beyers, 2011; Kilpatrick et al., 2001). These limitations indicate that Likert scale surveys can make it "difficult to assess ... [a respondent's] actual attitude toward a statement" (Kanak Pervez et al., 2020, p. 140).

Affective Spectrum Items

Another instrument developed to assist researchers and educators in accessing and assessing the mathematical dispositions of younger learners was developed by Graven in 2012. In contrast to a Likert-scale survey, Graven (2012) asked 10 Grade 3 students to identify where they felt like they were on a spectrum from weakest to strongest and thereby access how a student saw (or didn't see) themselves as an "effective learner and doer of mathematics" (p. 53; cf. Kilpatrick et al., 2001). In addition to providing a neutral option for students on this spectrum, Graven's (2012) instrument also included several additional writing prompts and an interview component. According to Graven (2012), these writing prompts and interviews provided researchers with "rich textured utterances on how learners perceived productive (and unproductive) learning dispositions" while also providing researchers with opportunities to access and assess even more components of students' mathematical dispositions (Graven, 2012, p. 54). Specifically, Graven's (2012) instrument had the capability to examine whether students saw sense in mathematics, perceived mathematics as useful and worthwhile, believed effort paid off, and if students saw themselves as effective learners and doers of mathematics (cf. Kilpatrick et al., 2001). Aspects of this instrument promise richer descriptions of students' mathematical disposition than Likert-scale items and may offer further promising results with students in additional grades.

Student-Generated Metaphor Items

Other researchers leveraged metaphors to access or assess components of students' mathematical disposition. According to Lakoff and Johnson (2003), "The essence of metaphor is understanding and experiencing one kind of thing in terms of another" (p. 5). A metaphor can be understood simply as a "comparison between two things, based on resemblance or similarities" (Cai et al., 2012, para. 3). Most often these similarities are non-literal in nature (Dogan et al., 2019; Gentner, 1982) and tend to highlight a relational similarity between two objects or concepts (Gentner & Smith, 2012). In this manner, information from the more familiar object (i.e., source or base) is mapped onto the less familiar object (i.e., target; Chou & Shu, 2015; Gentner & Smith, 2012).

We focused our review of the literature on how researchers used metaphors to examine PK through high school (i.e., aged 4–18) students' views of mathematics and have identified several key observations. For example, researchers have used various structures when designing student-generated metaphor item instruments to study students' mathematical dispositions. Some researchers asked students to create their own metaphors for mathematics (e.g., Schinck et al., 2010) or the learning of mathematics (e.g., Güner, 2012, 2013) without any constraints. When using these instruments, students could choose the source or base of the metaphor for comparison. In other studies (e.g., Cai & Merlino, 2011; Cai et al., 2012; Taing et al., 2015), researchers asked students to complete a metaphor already started for them. This structure is often called a metaphor starter because the source or base of the metaphor for comparison is pre-selected. For example, Cai and Merlino (2011) asked high school students to complete the following: "If math were an animal, it would be [blank for student response], because [blank for student response]" (p. 148). Other researchers used metaphor starters relating mathematics to food, colors, or animals—Taing et al. (2015) with middle school students (i.e., Grade 6) and Cai et al. (2012) with high school students (i.e., Grades 9–12).

Despite the variety in structure of metaphor instruments, Cai and Merlino (2011), Cai et al. (2012), Güner (2012), and Taing et al. (2015) all discussed how students' metaphors indicated a variety of feelings toward mathematics. Specifically, Cai and Merlino (2011) and Cai et al. (2012) categorized responses along an affective spectrum spanning from very negative to very positive feelings about mathematics. In contrast, Güner (2012) and Taing et al. (2015) classified metaphors into more concrete levels or categories. Specifically, Güner (2012) classified metaphors as indicating either positive, negative, or neutral attitudes toward mathematics. In contrast, Taing et al. (2015) identified two levels of enjoyment (i.e., "like/love" and "variable enjoyment") and reported that "no metaphors displayed an absolute and definitive dislike for mathematics" (p. 237).

In addition to examining feelings toward mathematics, Cai and Merlino (2011), Cai et al. (2012), Güner (2012), Schinck et al. (2010), and Taing et al. (2015) described tangential themes from their coding. Cai and Merlino (2011) discussed themes related to why or how students felt as they did about mathematics, and Taing et al. (2015) related students' feelings to motivation and engagement. Interestingly, Güner (2013) and Schinck et al. (2010) identified categories and themes that went beyond the affective spectrum. Güner (2013) identified eight themes: "(1) discovering an unknown, (2) learning a new skill, (3) solving a puzzle, (4) learning the rules and playing a game, (5) using a tool, (6) difficulties of learning mathematics, (7) pleasure of learning mathematics, (8) having a hardship" (p. 1947). In contrast, Schinck et al. (2010) identified five themes: perseverance (i.e., math is challenging, requires effort, and is rewarding), structure (i.e., math as an interconnected structure or hierarchical structure), journey (i.e., math is an enjoyable or uncertain journey that requires effort), tool (i.e., math is useful), and student role. These themes indicate that mathematical disposition is not just affective in nature, but rather that the full breadth and complexity of students' mathematical disposition is not yet clear.

Results from these six studies (i.e., Cai & Merlino, 2011; Cai et al., 2012; Güner, 2012, 2013; Schinck et al., 2010; Taing et al., 2015) indicate middle and high school students have "complex but well-developed views about mathematics" (Schinck et al., 2010, p. 332). Although some researchers have argued that using metaphors is only appropriate with older students because they need to be able to think relationally and figuratively about concepts (e.g., Cai et al., 2012; Taing et al., 2015), others have argued that young children's ability to think metaphorically may be underestimated (Billow, 1975; Gentner, 1988; Vosniadou, 1987). Further, because understanding or interpreting metaphors is more difficult or cognitively demanding than producing metaphors (cf. Gentner, 1988), we conjecture that researchers may be able to use metaphor starters to assess and access components of their mathematical disposition with students before they are in middle school.

Summary

Although the current literature offers a few ways in which researchers and teachers can access and assess older students' mathematical dispositions, less is known about components of younger students' mathematical disposition (i.e., PK – Grade 3). Further, previous research has not yet captured the full breadth and complexity of variation in older students' mathematical disposition (i.e., Grades 4–8). Hence, we posed the following research question:

> What components of mathematical disposition are revealed when PK through Grade 8 students use metaphors to describe mathematics?

Methods

We developed and administered a survey to students enrolled in a PK-Grade 8 school during the spring of 2023. This study took place at a small public school (i.e., school population was 424 students during the 2022–2023 school year) in a suburban community located in the Midwestern region of the United States. During the 2022–2023 school year, 79% of the student population identified as White/nonHispanic, 10% as Hispanic, 3% as Black/nonHispanic, 8% as Asian, and <1% as American Indian/Alaskan. The school's student population reflects the community's demographics.

Survey Instrument and Data Collection

We examined PK-Grade 8 student responses to a metaphor starter relating food to mathematics. We chose this metaphor starter to build off the work of Cai et al. (2012) and because food was more accessible than other metaphor sources (e.g., animals). For PK, kindergarten (K), and Grade 1 students, researchers conducted individual interviews and verbally asked students to finish the following prompt: If math were a food, it would be ... (because ...). Each student response to this prompt was entered into Qualtrics in real time, and data were later de-identified. For Grades 2–8, students anonymously provided a written response to the questions: If math were a food, what food would it be? Why would it be that food?

Data Analysis

We engaged in five phases of data analysis. During Phase 1, Authors 2 and 3 took up an open coding scheme by independently reading each participant's response ($n = 306$), our unit of analysis. Specifically, we segmented these responses into meaningful parts and coded each of these parts to generate a list of initial mathematical disposition codes (Saldaña, 2021). For instance, we segmented the following singular response into three meaningful parts: Math would be "Pasta [because] there are so many intertwining strands of knowledge," "with little 'meatballs' of theorems." "You also can't eat too much at a time, you have to pace yourself."[1] We also assigned the following initial codes to this response: complexity of mathematics, specific topic in mathematics, and time commitment, respectively.

Next, in Phase 2, Authors 2 and 3 met to compare and sort their initial codes, create a unified list, and write initial definitions for each code. This list of initial codes and definitions was then used to independently recode all participant responses. As Authors 2 and 3 compared their response codings, all disagreements were negotiated and resolved. Sometimes these negotiations involved collapsing codes, and other times these negotiations involved the creation of new codes. These negotiations allowed for the continual revision and refinement of codes and definitions.

In Phase 3, Authors 2 and 3 trained Author 1 on the coding scheme. Author 1 then independently coded 20% of the responses, which falls within the typical range of 10–25% (O'Connor & Joffe, 2020). When all three researchers compared Author 1's coding with the coding of Authors 2 and 3 from Phase 2, a Cohen's kappa score of 0.80 was achieved. According to Landis and Koch (1977), a kappa statistic between 0.61 and 0.80 indicates substantial agreement. To attain stronger agreement, Authors 1–3 met in Phase 4 to resolve any outstanding disagreements, discuss component codes, and clarify definitions. The three researchers then independently recoded all responses to create a final set of codes and code definitions, resulting in a Fleiss kappa statistic of 0.93 of agreement, which is considered near perfect agreement (Landis & Koch, 1977).

During Phase 5, Authors 1–3 returned to the conceptual framework to consider each code individually. Using our code definitions, we determined that most codes could be categorized as a component of one of the three mental functions outlined by Beyers (2011). In terms of the pasta example, the final component codes assigned were Specific Content or Component of Mathematics, Variety and Complexity of Math, and Time Element, which indicated both cognitive and conative mental functions (see Figure 1). However, we found that our definitions for two component codes, Other and Outcome Focus, could span more than one mental function. For those component codes, we determined which mental function was indicated based on the essence of the individual response.

Of the 306 participants in this study (see Table 1), 50 participants did not create a metaphor. Hence, we attempted to code responses from 256 participants. In the end, only eight metaphors were not coded using our coding scheme and are excluded in the results below. Two such examples include "Paper because I eat paper like napkins but not anymore" and "Matharoni. Macaroni and add some math." Below we discuss our findings from the remaining 248 responses that we coded and then categorized using Beyers (2011) framework.

[1]We changed spelling to improve readability when it was clear to Authors 1–3 which word was intended, but we did not adjust sentence structure or phrasing of student responses.

	Code	Definition	Example
Affective	Ability to Learn Mathematics	Indicating beliefs about learning mathematics	Grapefruit. You will spend some time trying to figure it out and then it will become easy like math.
	Applicability of Mathematics	Indicating beliefs about the utility of mathematics	Pie because we will use pi in other grades.
	Mathematics is Foundational	Indicating beliefs about the fundamental value of learning mathematics	Meats made of protein, which is the building block of life. Math is sort of the building block of everything.
	Enjoyment of Mathematics— Conditional Enjoyment Negative Enjoyment Positive Enjoyment Others' Enjoyment	Indicating one's (conditional, negative, or positive) or others' attitudes or feelings about mathematics	One's Enjoyment: Spicy food. Because sometimes I like it and sometimes, I despise it. (Conditional) A strawberry because I hate strawberries. (Negative) Cupcakes - because I like cupcakes and math. (Positive) Others' Enjoyment: Brussel sprouts because most people don´t like it.
	Prevalence of Mathematics	Indicating versatility of mathematics	Leftovers. When you think it's done, there's always more.
Cognitive	Outcome Focused	Indicating a focus on outcome or result	Crepes...the results are worth it.
	Processes and Approaches Used in Mathematics	Indicating variety in ways to solve or do mathematics	Cauliflower because depending on how you cook it, it tastes different just like different math problems.
	Specific Content or Component of Mathematics	Indicating specific concepts, topics, or ideas of mathematics	Pizza and pie because if you slice it up you could make a fraction.
	Variety and Complexity of Math	Indicating connections among various aspects, topics, or components of mathematics	Pickles because pickles have complex flavors and a deep meaning. not just what's one the surface. so does math I think.
Conative	Outcome Focused	Indicating a focus on outcome or result	A hamburger...[different ingredients] to find a different outcome.
	Level of Challenge	Indicating the mental work or effort it takes to learn, understand, or do mathematics	Ramen noodles because it would be hard to untangle it.
	Time Element	Indicating the amount of time it takes to learn, understand, or do mathematics or how one's experience with mathematics changes over time	Watermelon. It takes a long time to prepare, but there are many ways to use it when finished.
	Outcome Focused	Indicating a focus on outcome or result	A lemon [making lemonade] is like you kind of solved the problem...

Figure 1. Final code definitions per mental function.

Table 1. Distribution of Student metaphors.

		Metaphors		
Grade	Number of Students	Coded	Not Coded	Blank or Not Created
Pre-Kindergarten	22	4	2	16
Kindergarten	26	14	1	11
1	36	28	4	4
2	22	14	0	8
3	23	21	0	2
4	12	12	0	0
5	42	40	1	1
6	48	43	0	5
7	43	41	0	2
8	32	31	0	1
Total	306	248	8	50

Results

We categorized the majority of the 248 student responses (71%) as affective mental functioning. We also categorized about one-fourth (28%) of student responses as cognitive mental functioning and almost one-sixth (16%) as conative mental functioning. Because a student's response may reflect multiple components of mathematical disposition, some responses indicated more than one mental function.

Affective Mental Function Components

We classified seven components as reflecting affective mental functions. These seven components pertain to student feelings, attitudes, and beliefs about the learning or doing of mathematics as well as the nature of mathematics (i.e., its relevance or usefulness). In total, 177 student responses included at least one affective mental function component. Table 2 shows the distribution of responses.

Table 2. Affective function code frequency.

Code	*n*	%
Enjoyment of Mathematics	128	72.3
Conditional Enjoyment	22	
Negative Enjoyment	28	
Positive Enjoyment	55	
Others' Enjoyment	23	
Mathematics is Foundational	16	9.0
Applicability of Mathematics	9	5.1
Outcome Focus–Affective	9	5.1
Ability to Learn Mathematics	10	5.6
Prevalence of Mathematics	4	2.3
Other–Affective	25	14.1

$N = 177$. Sum of percentages exceeds 100% since a response can contain more than one affective function.

Enjoyment of Mathematics

We identified approximately 72% of affectively categorized student responses as reflecting Enjoyment of Mathematics. Some of the participants' responses indicated that their enjoyment varied in some ways (i.e., Enjoyment of Mathematics – Conditional). For example, a fifth-grade student wrote, "Chocolate because it is good and fun but if you do too much of it or eat too much it is not fun anymore." Some of the participants responded in a way that referred to others' attitudes or feelings rather than explicitly indicating their own enjoyment (i.e., Enjoyment of Mathematics – Others).[2] For example, a sixth-grade student said math would be, "Lemons because some people love it and think that lemons are sweet, but some people think that lemons are sour and don't like them." Participants whose responses indicated a positive or negative enjoyment of mathematics also tended to indicate the extent to which they positively or negatively enjoyed mathematics. For example, we note a difference in the extent of negative enjoyment when juxtaposing the responses: "Peas are my least favorite food" (Grade 5 student), and "A flaming cactus covered in dirt and glass because math is painful" (Grade 8 student).

Additional Affectively Categorized Components

More than 40% of affectively categorized student responses reflected the remaining six components. Specifically, 9% of these responses indicated a sense that mathematics was important or vital (i.e., Mathematics is Foundational). A sixth-grade student, for example, wrote that math would be "A carrot [because] carrots are good for you [because] bones and math is kinda like the structure of your mind." An additional 6% of student responses reflected a perceived ability to learn (i.e., Ability to Learn Mathematics). A first grader said if math were a food, it would be "Yummy – because you will get smarter and smarter, and you would be the smartest person in the world."

Approximately 5% of student responses referred to when mathematics is used or applied. For instance, a fifth-grade student responded that math would be pi "because we will use pi in other grades," which indicated the component Applicability of Mathematics. Yet another 5% of student responses indicated a focus on mathematical outcomes in affective ways (i.e., Outcome Focus – Affective). Take, for example, the eighth-grade student who wrote, "All jokes aside, a pie is a really good representation of math because it shows that if you learn it right, it can turn out great, but if you learn something wrong it can be bad (burning the apples)." Only 2% of responses indicated an affective function that referred to the Prevalence of Mathematics, such as the Grade 8 student who said, "when you think it's done, there's always more."

[2]We determined that some responses clearly indicated the participants' personal feelings or attitudes toward mathematics, and we were able to code those responses as Enjoyment of Mathematics – Positive, Negative, or Conditional. However, other responses were murkier. Participants' phrasing made it unclear how the individual personally felt about mathematics. Hence, we coded such responses as Enjoyment of Mathematics – Others to capture this difference.

For the remaining 14% of student responses categorized as affective, we were unable to identify one of the existing seven components (i.e., Other – Affective). Take, for example, the eighth-grade student who wrote, if math were a food, it would be dirt because "There is a lot of gray areas" or the PK student who said it would be "a clown pancake because it's funny." Both responses indicate feelings these students experience with mathematics; however, what the eighth-grade student meant about gray areas and what is funny about mathematics for the PK student are both unclear.

Affective Grade-Based Comparisons

When we compared Enjoyment of Mathematics subcomponents across grade levels (see Table 3), we noticed that a greater proportion of older participants' metaphorical responses reflected Enjoyment of Mathematics – Negative more than the younger participants. Specifically, of the responses that indicated Enjoyment of Mathematics – Negative, 81% came from participants in Grades 5–8. Similarly, of the responses that indicated Enjoyment of Mathematics – Positive, 76% came from PK – Grade 4. We also noticed that none of the younger participants' responses indicated Enjoyment of Mathematics – Conditional, and only 12% of younger participants referred to Enjoyment of Mathematics – Others.

When comparing five affectively categorized components (excluding Enjoyment of Mathematics and Other – Affective) across grade levels (see Table 4), we noticed that none of the PK and K students' responses reflected these components. Similarly, we noticed that none of the younger participants' metaphors referred to the Applicability of Mathematics. We also noticed that all the responses reflecting the Prevalence of Mathematics were produced by older participants in Grades 6 and 8.

Table 3. Proportion of enjoyment type per grade.

	Enjoyment of Mathematics							
	Positive		Negative		Conditional		Others'	
Grade	*n*	%	*n*	%	*n*	%	*n*	%
PK	1	1.8	0	0.0	0	0.0	0	0.0
K	9	16.4	1	3.6	0	0.0	1	4.3
1	15	27.3	1	3.6	0	0.0	0	0.0
2	9	16.4	2	7.1	0	0.0	0	0.0
3	6	10.9	0	0.0	0	0.0	1	4.3
4	2	3.6	1	3.6	0	0.0	1	4.3
5	4	7.3	8	28.6	5	22.7	0	0.0
6	5	9.1	5	17.9	8	36.4	7	30.4
7	3	5.5	6	21.4	6	27.3	7	30.4
8	1	1.8	4	14.3	3	13.6	6	26.1

Table 4. Proportion of non-enjoyment affective components per grade.

	Mathematics is Foundational		Ability to Learn Mathematics		Applicability of Mathematics		Prevalence of Mathematics		Outcome Focused– Affective	
Grade	*n*	%	*n*	%	*n*	%	*n*	%	*n*	%
PK	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
K	0	0.0	0	0.0	0	0.0	0	0.0	0	0.0
1	4	25.0	2	20.0	0	0.0	0	0.0	0	0.0
2	0	0.0	1	10.0	0	0.0	0	0.0	0	0.0
3	1	6.3	2	20.0	0	0.0	0	0.0	1	11.1
4	0	0.0	0	0.0	1	11.1	0	0.0	0	0.0
5	4	25.0	1	10.0	2	22.2	0	0.0	3	33.3
6	1	6.3	0	0.0	2	22.2	1	25.0	0	0.0
7	4	25.0	2	20.0	0	0.0	0	0.0	2	22.2
8	2	12.5	2	20.0	4	44.4	3	75.0	3	33.3

Cognitive Mental Function Components

We categorized five components as indicating cognitive mental functions. The components in this category referred to specific mathematical symbols, concepts, skills, or processes, and included reasoning, making arguments, following steps, or solving problems. We identified 69 responses that reflected at least one cognitive mental function component, Table 5 shows the distribution of responses.

We found almost half of the responses categorized as a cognitive mental function referred to specific symbols, concepts, numbers, or operations (i.e., Specific Content or Component of Mathematics). For example, a first-grade student noted that may would be, "Number cookies because math is all about numbers," and a Grade 4 student wrote math would be cookie bars because "It's easier to do fractions with a rectangle." About 40% of the responses categorized as a cognitive mental function focused on the Variety and Complexity of Mathematics. For example, a seventh-grade student asserted that if math were a food, it would be grapes because "there are the basics which would be the vine connecting everything but the grapes would be different types of problems."

Another 12% of student responses referred to the Processes and Approaches of Mathematics. This fifth-grader's response: "Rice has many different ways to eat it and like math you can solve it in many different ways" is one such example. Finally, 3% of student responses indicated a focus on outcomes in cognitive ways (i.e., Outcome Focus – Cognitive), and another 4% of responses referred to an indeterminant cognitive function (i.e., Other – Cognitive).

Table 5. Cognitive function code frequency.

Code	*n*	%
Specific Content or Component of Mathematics	34	49.3
Variety and Complexity of Mathematics	28	40.6
Process and Approaches of Mathematics	8	11.6
Outcome Focus–Cognitive	2	2.9
Other–Cognitive	3	4.3

N = 69. Sum of percentages exceeds 100% because a response can contain more than one cognitive function.

Cognitive Component Comparisons

When we compared these five cognitively categorized components across grade levels, several trends emerged (see Table 6). Students from nearly all grade bands (K through Grade 8) produced metaphors that referred to Specific Content or Component of Mathematics. In contrast, only older participants' (i.e., Grades 5–8) metaphors referred to Variety and Complexity of Mathematics. Similarly, most metaphors reflecting the components Process and Approaches of Mathematics and Outcome Focus – Cognitive were produced by participants older than Grade 2.

Table 6. Proportion of cognitive components per grade.

	Specific Mathematical Content		Variety and Complexity of Mathematics		Processes and Approaches of Mathematics		Outcome Focused–Cognitive	
Grade	*n*	%	*n*	%	*n*	%	*n*	%
PK	0	0.0	0	0.0	0	0.0	0	0.0
K	1	2.9	0	0.0	0	0.0	0	0.0
1	3	8.8	0	0.0	0	0.0	0	0.0
2	0	0.0	0	0.0	0	0.0	0	0.0
3	9	26.5	0	0.0	1	12.5	1	50.0
4	6	17.6	0	0.0	0	0.0	0	0.0
5	7	20.6	6	21.4	5	62.5	1	50.0
6	4	11.8	4	14.3	1	12.5	0	0.0
7	3	8.8	7	25.0	0	0.0	0	0.0
8	1	2.9	11	39.3	1	12.5	0	0.0

Conative Mental Function Components

We categorized three components as indicating conative mental functions. These components emphasized "diligence, effort, or persistence in the face of mathematical activity" (Beyers, 2011, p. 72). We identified 39 responses that reflected at least one conative mental function component. Table 7 shows the distribution of responses.

More than 80% of the responses categorized as a conative mental function referred to challenges or difficulties associated with mathematics (i.e., Level of Challenge). For example, a sixth-grade student wrote, it would be "a complicated dish like a souffle, because just like math, there are a lot of different things you need to do, and a lot of different things that could go wrong." Furthermore, about 20% of student responses referred to the time it takes to learn, understand, or do mathematics (i.e., Time Element). Finally, approximately 3% of student responses indicated a focus on outcomes in conative ways (i.e., Outcome Focus – Conative). Similar to results discussed with cognitive mental functions, these latter two components always occurred in combination with other components, which will be discussed in the section on multiple mental functions.

Table 7. Conative function code frequency.

Code	n	%
Level of Challenge	32	82.1
Time Element	8	20.1
Outcome Focused–Conative	1	2.6
Other–Conative	5	12.8

N = 39. Sum of percentages exceeds 100% because a response can contain more than one Conative disposition.

Conative Component Comparisons

As for grade-based trends pertaining to conative components (see Table 8), we identified trends similar to those we reported for cognitive components. For example, much like Specific Content or Component of Mathematics, students PK – Grade 8 produced metaphors that referred to Level of Challenge. However, only older participants' (i.e., Grades 5–8) metaphors referred to Time Elements.

Table 8. Proportion of conative components per grade.

	Level of Challenge		Time Element		Outcome Focused–Conative	
Grade	*n*	%	*n*	%	n	%
PK	1	3.1	0	0.0	0	0.0
K	1	3.1	0	0.0	0	0.0
1	2	6.3	0	0.0	0	0.0
2	1	3.1	0	0.0	0	0.0
3	4	12.5	2	25.0	0	0.0
4	1	3.1	0	0.0	0	0.0
5	2	6.3	1	12.5	0	0.0
6	9	28.1	3	25.0	1	100.0
7	8	25.0	0	0.0	0	0.0
8	3	9.4	3	37.5	0	0.0

Multiple Mental Functions

Although many student responses illustrated solely affective, cognitive, or conative mental functioning, 36 responses indicated a combination of two or more mental functions (see Figure 2). We categorized 15 of these responses as both affective and cognitive, 17 as both affective and conative, three responses as both cognitive and conative, and one response as all three mental functions (see Figure 3). The response indicating all three mental functions was math would be "Noodles [because]

Components	Response	Codes
Multiple Affective Components	Candy because it's good and math is good because it helps you learn.	Enjoyment of Mathematics–Positive Mathematics is Foundational
	Salmon. I don't like salmon but I eat it because it's good for you	Enjoyment of Mathematics–Negative Mathematics is Foundational
Multiple Cognitive Components	Pizza. You can change you way of how you do math like changing to a different type of pizza you can split math into many parts like pizza.	Processes and Approaches Used in Mathematics Variety and Complexity of Mathematics
	If math was a food, it would be a hamburger. Math is still one thing in general, meat with bread and a topping of some sort, but it can be looked at with different approaches and types of math, different flavors/ toppings, to find a different outcome.	Variety and Complexity of Mathematics Processes and Approaches Used in Mathematics Outcome Focus–Cognitive
Multiple Conative Components	Chocolate hard at first but gets easy to chew after a while.	Level of Challenge Time Element
	Ice cream because at first it's hard but once you set it out for a little bit it gets easier.	Level of Challenge Time Element
Affective and Cognitive Components	Pizza because I love pizza and it can be cut into fractions.	Enjoyment of Mathematics – Positive Specific Content or Component of Mathematics
Affective and Conative Components	An avocado because when they are not ripe it is hard to eat because it would not taste good but if it is it is easy to eat because it taste really good.	Enjoyment of Mathematics – Conditional Level of Challenge

Figure 2. Responses indicating multiple affective, cognitive or conative functions. *Note*. Codes are not necessarily listed in the order functions are listed or segments were coded.

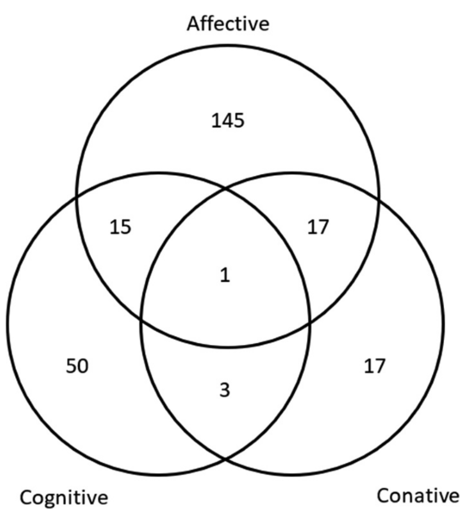

Figure 3. Venn diagram of Student responses across affective, cognitive, and conative functions.

they always slip through your fork, and you can't get them like how math problems don't always work out right." This metaphor reflected Ability to Learn (an affective mental function), Level of Challenge (conative mental function), and Variety and Complexity of Mathematics (a cognitive mental function).

Discussion

This study's purpose was to access and assess components of mathematical disposition when PK through Grade 8 students use metaphors to describe mathematics. Notably, our findings in this study: (1) align with several themes, categories, and codes previously identified in literature; (2) expand on those from others in the field by revealing that mathematical disposition is not only complex but also conditional; and (3) expand on additional existing research by revealing that components of mathematical disposition may trend in relation to grade bands.

Alignment of Themes, Categories, and Codes

Although some studies on students' mathematical dispositions lack clear descriptions for all their themes, categories, or codes, several notable comparisons between our codes and those utilized in other studies can be productively juxtaposed. In total, we identified seven affective components, five cognitive components, and three conative components. Of the seven affective components, five were productively juxtaposed with findings of other researchers including Schinck et al. (2010), Taing et al. (2015), and Cai et al. (2012). For example, our component Ability to Learn Mathematics appears to be relatively consistent with Schinck et al.'s (2010) themes of "Perseverance" and "Student Role" in recognizing that math requires effort and that students must take an active role in their learning (p. 329). In a similar manner, our component Applicability of Mathematics was compared to Schinck et al.'s (2010) theme "Tool," our component Mathematics is Foundational was compared to Taing et al.'s (2015) sub-category "Valuing of Mathematics," the components Enjoyment of Mathematics – Positive and Enjoyment of Mathematics – Negative were compared to Cai et al.'s (2012) and Taing et al.'s (2015) views of enjoyment of mathematics, and our component Outcome Focus – Affective was compared to Schinck et al.'s (2010) sub-themes of "Math is Rewarding" and "Journey of Discovery."

Of the five cognitive components, two were productively juxtaposed with the findings of Schinck et al. (2010) and Cai and Merlino (2011). For example, our component of Variety and Complexity of Mathematics appears to be consistent with Schinck et al.'s (2010) "Structure" theme as well as their "Interconnected" sub-themes (p. 329). Similarly, our component of Specific Content or Component of Mathematics aligns with Cai and Merlino's (2011) study in which student reasoning referenced specific mathematical content. Processes and Approaches Used in Mathematics, Outcome Focus – Cognitive, and Other – Cognitive were not clearly aligned with findings of other researchers.

Of the four conative components, two were productively juxtaposed with the findings of Schinck et al. (2010), Cai and Merlino (2011), and Güner (2013). The most prominent component in the conative category was Level of Challenge, which aligns with all three studies and specifically, Güner's (2013) categories of "Solving a Puzzle" or "Difficulties of Learning Mathematics" (p. 1948). Similarly, our component Time Element can be compared to Schinck et al.'s (2010) theme of "Journey" (p. 329). By contrast, Other – Conative and Outcome Focus – Conative did not align productively with themes in the literature. Other and Outcome Focus are certainly unique and complex components as they span all three mental function categories outlined by Beyers (2011).

Complexities and Conditionality of Mathematical Disposition

In alignment with Schinck et al.'s (2010) belief that the categorization of metaphors into a singular theme fails to properly contextualize student metaphors, we created and identified multiple disposition components for further investigation with respect to each mental function (i.e., affective, cognitive, or conative). We also allowed components to span multiple categories of mental functions. This method of examining and coding segments of students' responses proved useful in highlighting the complexities of mathematical disposition.

With respect to affective mental functioning, we identified and defined seven components. One of these components, Enjoyment of Mathematics, was then further divided into four subcomponents related to *how* that enjoyment was expressed. Like researchers Cai et al. (2012) and Taing et al. (2011), we noted that most of the metaphors reflected positive or negative enjoyment of mathematics (i.e., subcomponents Enjoyment of Mathematics – Positive and Enjoyment of Mathematics – Negative). In addition to these two expressions of enjoyment, however, we also identified metaphors that revealed *conditional* enjoyment of mathematics (i.e., Enjoyment of Mathematics – Conditional), when a student indicated that they may have more positive feelings about mathematics during one moment in time but less positive feelings at another. Our component code Enjoyment of Mathematics – Conditional also indicated that a student may value mathematics or experience struggle in one context differently than in another.

Further extending the existing body of evidence was the emergence of metaphors that referred to *others'* enjoyment of mathematics (i.e., Enjoyment of Mathematics – Others, our fourth subcomponent). Several student responses referred to some people liking mathematics and others not liking mathematics. Several more student responses referred to others in ways that caused researchers to infer the student's own sentiments. One student used the phrase "bitter nobody likes it" which caused researchers to infer that the student completing the metaphor also did not like math. This important distinction between self and others resulted in a fourth subcomponent, Enjoyment of Mathematics – Others. This unintentional surfacing of others' feelings toward mathematics in our study seems to align with Graven's (2012) decision to intentionally ask students to articulate views of themselves in terms of other learners. Perhaps, like Graven argued in her study, students in our study felt safer, or more comfortable, expressing components of their own mathematical disposition in terms of the mathematical dispositions of others.

Grade-Based Trends

As mentioned throughout our results section, a closer examination of components across grade levels revealed several notable trends. First, the depth and breadth of detail provided by some of the metaphors used by many of the younger students in our study were narrower than the older students. Furthermore, their metaphors also tended to be more literal in nature.

Second, grade-based trends also highlighted a clear divide between certain grade bands for a particular component. For example, younger students in PK through Grade 4 clearly held the largest proportion of Enjoyment of Mathematics – Positive components. In contrast, older students in Grades 5–8 clearly held the largest proportion of Enjoyment of Mathematics – Negative component. Findings from prior studies conducted with older students (e.g., Güner, 2013) indicate a major shift in mathematics content and experiences may be part of the reason for this shift in enjoyment-related affective components. Based on conversations with school personnel at the study site, it is our conjecture that this shift is a result of a change in rigor and classroom structure. At this school, Grades 5–8 are considered middle school and subjects are departmentalized, whereas PK – Grade 4 are self-contained classrooms.

Third, grade-based trends often highlighted a saturation of components for certain grade bands. For example, the only affectively categorized component exhibited in metaphors used by PK and

K students was Enjoyment of Mathematics. On the other end of the grade level spectrum, only older participants used metaphors to describe the Applicability of Mathematics, Prevalence, and Variety and Complexity of Mathematics. This may be due to older students having more life and school experiences with mathematics, thereby providing them with more opportunities to see the usefulness, pervasiveness, or complexity of mathematics than younger students. Yet another code occurring predominately in responses of Grade 5–8 students was Time Element. In fact, no students in PK to Grade 2 mentioned Time Element in their metaphorical responses. The remaining four components spanned PK through Grade 8.

Study Limitations

This study provided a platform for students from a wide range of ages, grades, and therefore levels of cognitive maturity to utilize metaphors as a means of accessing and describing their mathematical disposition. As conjectured, we were able to use metaphor starters to assess and access components of students' mathematical disposition before middle school. That said, although PK and K students were included in our study sample, we could only code four of 22 PK metaphors and 14 of 26 K metaphors. It was not until Grade 3 (i.e., approximately ages 8–9) that more than 90% of student-generated metaphors were codable. Thus, it is reasonable to acknowledge that the metaphorical reasoning of younger students included in this study may not be as effective or productive in expressing their mathematical disposition as the reasoning of older students. However, it is nonetheless important to recognize the emergence of these higher-order thinking skills in even our youngest students.

Another limitation of the study pertains to the interpretation of student responses. For example, some terms used by students like the word "funny" could be interpreted as silly and goofy or as strange and unusual. It would have been helpful in the case of several student responses to have the opportunity to follow up with clarifying questions. Without the opportunity for students to further elaborate on or explain their responses there is a possibility of misinterpreting the relational intent of a metaphor.

Implications for Researchers and Practitioners

In reflecting on key findings from this study, we offer several implications for mathematics education researchers and practitioners. We consider these implications in terms of our conceptual framework (i.e., the three categories of mental functions) and in terms of the grade bands included in this study.

First, our study demonstrates the value of focusing on *all* three categories of mental functions. As mentioned, 39 student responses attended to conative mental functions and 69 student responses attended to cognitive mental functions, and 36 responses attended to at least two mental functions. This contrasts to most prior research focused on only one mental function category (most frequently affective). And, as it specifically pertains to affective mental functions, our findings suggest that labeling a student's mathematical disposition as either positive or negative is an unproductive and overly simplified practice. Our findings reveal the often conditional and complex nature of students' enjoyment (or lack thereof) associated with mathematics.

Second, these findings suggest that researchers and practitioners should dig deeper into the complexities and nuances of individual students' mathematical disposition to determine what factors are influencing how students engage with mathematics. For example, a researcher or practitioner might be interested to know what classroom conditions, teaching practices, specific mathematical content may cause certain students to enjoy (or not enjoy) mathematics at that point in time. For example, knowing that a particular student relates math to peanut butter because "peanut butter is tough and hard to spread, but sometimes if you get the perfect amount, it can be good. Too much is just a lot to handle" may help a practitioner actively look for when that student may be feeling like the math is getting to be "too much" to handle.

Third, in terms of the developmental readiness of students to engage in metaphorical reasoning, our results provide additional evidence that some young children can reason with or about metaphors before Grade 3 (cf. Billow, 1975; Gentner, 1988; Vosniadou, 1987). Certainly, many of the metaphors utilized by PK – Grade 2 students were more literal in nature than those utilized by older students, but again, it is important to recognize the emergence of these higher-order thinking skills in even our youngest of students. For example, both a PK student and a Grade 5 student metaphorically reasoned about the challenges or difficulties with mathematics. The PK student stated that math is like an apple because "science is hard and an apple is hard," whereas the fifth-grade student stated that math is like Sour Patch Kids because "Sour Patch Kids are sour then sweet which reminds me of times when math is really hard and then becomes easy." We consider this PK student's metaphor to be more literal in nature, whereas the Grade 5 student's metaphor to be more relational in nature. We conjecture the literal nature of many of these metaphors may be due to a lack of the use of the term "math" or "mathematics" in the early grades, especially in PK and K. Teachers of students in the study sample indicated that they often referred to their math curriculum activities as number corner, calendar, center, or game time instead of explicitly math activities.

Finally, researchers and practitioners should pay attention to the presence and absence of specific components among various grade bands. It is possible that the strong presence of a given component may help researchers and practitioners understand which components of mathematical disposition are most influential on students of certain grade bands. For example, findings from our study revealed that the only affectively categorized component exhibited in metaphors used by PK and K students was Enjoyment of Mathematics. This indicates to researchers and practitioners that younger students' feelings toward mathematics should not be ignored and instead must be acknowledged and considered. Findings from our study also revealed that the Enjoyment of Mathematics – Negative component was most prominent with older students in Grades 5–8. This indicates to researchers and practitioners that attention should be given to promoting positive associations with mathematics with this population. Components that span multiple grade bands may also provide meaningful opportunities for researchers and practitioners to track and observe the development of mathematical disposition over time.

Future Research

The results of this study extended the literature to reveal how mathematical disposition is complex, especially in terms of its many components. However, we cannot say that the components identified in this study capture all possible components of mathematical disposition. Future research should examine whether other components exist as well as reexamine and extend definitions of currently identified components.

Additionally, further research needs to examine students' mathematical disposition in other schools or geographical locations. Students in new settings may link food and mathematics via additional components of mathematical dispositions not identified in this study. Similarly, research should examine the components of mathematical disposition revealed through the creation of other metaphors. While food is an accessible source for many students to metaphorically reason with, it may not fully allow for the expression of all components of a student's mathematical disposition. In other words, other sources may reveal additional components to provide a more complete picture of a student's mathematical disposition.

Disclosure Statement

No potential conflict of interest was reported by the author(s).

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