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Data-Driven Mathematical Modeling and Simulation of Migration Dynamics During the Russian-Ukrainian War

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Abstract
In this work, we employ a governing system of ordinary differential equations (ODEs) to create a mathematical model for getting insights into the dynamics of migration of Ukrainians evacuating due to war. A suitable assumption on coefficients of this model results in the well-known logistic growth. Additionally, stability analyses of equilibrium solutions for these ODEs are performed, and we employ parameter estimation techniques to identify coefficients using online datasets via both a least-squares approach as well as a physics informed neural network approach. Our findings indicate that over time, the daily influx of Ukrainian refugees to Poland stabilizes at a constant rate, represented by an asymptotically stable equilibrium solution.

Keywords: Russian-Ukrainian War, Social Dynamics, Mathematical Modeling, Parameter Estimation, Physics-Informed Neural Networks

1 Introduction

Russia's unprovoked, full-scale invasion of Ukraine, which unfolded in February of 2022, had a significant impact on Ukraine across various aspects, see [19, 16]. The conflict resulted in a devastating loss of life [9], both human and ecological [6], causing immense human suffering and trauma for Ukrainian civilians and military personnel as well [7]. Infrastructure and residential areas were severely damaged due to frequently massive missile attacks, leading to widespread displacement and an ongoing humanitarian crisis. The war had regional and geopolitical implications, heightening tensions between Ukraine and Russia, impacting regional and international relations. Power dynamics and balances were shifted, leading to increased global scrutiny and encouraging diplomatic efforts to resolve the war. Internationally, the war disrupted trade and investments [5], hindering economic growth and development. However, amidst these challenges, the conflict emphasized the resilience, bravery, and solidarity of the Ukrainian people as communities came together to support those who were affected by the war, where civil society played a crucial role in providing aid and assistance to those in need.

Research in computational mathematics, which comprises of modeling, analysis, simulation and computing has become the foundation for solving most multidisciplinary problems in science and engineering. These real world problems often involve complex dynamic interactions of multiple physical processes which presents a significant challenge, both in representing the physics involved and in handling the resulting coupled behavior. If the desire to predict and learn from the system is added to the picture, then the complexity increases even further. Hence, to capture the complete nature of the solution to the problem, a coupled multidisciplinary approach is essential.

A true mathematical modeling process starts by identifying and observing a situation in the real-world from multiple perspectives. This is followed by the modeling process that involves learning to ask questions, making reasonable assumptions, eliminating unwanted information, identifying suitable variables (either quantitatively or qualitatively), and understanding the constraints with which they will have to contend [17]. It is well known that many physical systems that involve rates of change can be described by differential equations. Thus, understanding the behavior of the solution to such equations is important for elucidating an actual physical problem. Here, we apply theoretical and computational mathematical approaches to evaluate a differential equation in a social context. Theory is needed to guide the performance and interpretation of the numerical technique, and computation is necessary to synthesize the results. Therefore, the solution methodology often involves formulating a mathematical model from a physical system and then being able

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to solve this model using analytical (exact) or numerical (approximate) approaches. In addition, it is also important to be able to validate models for known datasets through efficient parameter estimation approaches [15].

While there have been some research in the past five years on applying mathematics to economic modeling [8] or rational urban land use [13] in the Ukraine context, to our knowledge, the recent critical humanitarian issue of understanding the dynamics of Ukrainian refugees across the Polish border has not yet received rigorous mathematical investigation. In fact, recently there was a case study of Ukraine on agent-based framework to study forced migration [11] where they study the efficacy of a data-driven agent-based framework motivated by social and behavioral theory in predicting outflow of migrants as a result of conflict events during the initial phase of the Ukraine war.

Our project aims to mathematically model and simulate the movement of Ukrainian refugees across the Polish border due to the ongoing war. We will model the nonlinear dynamics of two groups of populations using a system of differential equations leading to an initial value problem. Upon simplification of the model to the logistic growth, we are able to find an analytic solution to this simplified model. Following this, we conduct a analysis to determine equilibrium solutions and their stability. Subsequently, we perform a parameter estimation study to determine the coefficients of the proposed model, utilizing observed data extracted from the Polish Boarder Guard page using a traditional least-squares approach. With realistic parameter values derived from this analysis, our model predicts that, in the long term, the number of Ukrainian refugees evacuated to Poland will tend to stabilize at a constant value, represented by an asymptotically stable equilibrium solution.

Another innovative contribution in this paper is applying Physics Informed Neural Networks or PINNs [14, 15, 12]. The PINNs approach has been shown to be very efficient and robust approach for parameter estimation. So, we introduce the PINNs method in this work and apply it to the same data-set that we applied least squares method to identify the parameters in the model. Our results using both methods (least square fit and PINNs) are in good agreement, allowing us to capture the problem's parameters more precisely.

This paper is structured as follows: In Section 2, we discuss a social dynamics-based model governing the number of Ukrainian evacuees to Poland. We present the model's exact solution in the implicit form and conduct a stability analysis in Section 3. Section 4 covers parameter estimation for real data from the Polish Border Guard. Both the Least-Squares method and the PINNs framework are presented in this section that help to extract the parameters. Finally, Section 5 contains discussions and conclusions.

2 Mathematical Modeling of Migrations of Ukrainians

Various mathematical models, particularly those based on initial value problems for ODEs, can effectively describe the migration of evacuated Ukrainian refugees to Poland. In this context, we will specifically explore a class of models that help to capture the **social dynamics**, which are used to analyze interactions and changes within social systems over time (see e.g. [18]). Social dynamics models employ ODEs to depict how different variables or components of a social system evolve based on their interconnections and dependencies. In our investigation, these variables represent various segments of the Ukrainian population migrating from their home country (Ukraine) to a host country (e.g., Poland) due to the ongoing war.

We denote by $u_i(t)$ the number of Ukrainian residents in country i at time t, measured in days, where subscript 1 corresponds to Ukraine (the home country) and subscript 2 refers to Poland (the host country). Additionally, we introduce $\beta^{\pm}(t)$ as the transient probability per unit time for a migrating Ukrainian evacuee between the host and home countries. The social dynamics model is formally described as follows

$$\begin{cases} u_1'(t) = \beta^+(t)\, u_2(t) - \beta^-(t)\, u_1(t), \\ u_2'(t) = \beta^-(t)\, u_1(t) - \beta^+(t)\, u_2(t), \end{cases} \tag{1}$$

where a *closed system*, for which $u_1(t) + u_2(t) = 2U_0$, is assumed, where U_0 is the half of the current population of Ukraine (taken from February 2022). Making change of variable $u(t) := u_1(t) - U_0 = U_0 - u_2(t)$, we obtain

$$\begin{cases} u'(t) = \beta^+(t)\,(U_0 - u(t)) - \beta^-(t)\,(U_0 + u(t)), \\ u(0) = u_0. \end{cases} \tag{2}$$

Variable $u(t)$, measured in # of people, represents the deviation from an equal distribution of Ukrainians between the two countries (Ukraine and Poland) and measures how far the system is from a balanced state where both countries have exactly half of the total population of Ukrainians.

Once again, one can select different models for the transient probabilities $\beta^{\pm}(t)$, (measured in 1/day). We assume that β^+ is directly proportional to the number of Ukrainian refugees moving to Poland. Since we do not consider the reverse movement of Ukrainians, we set $\beta^-(t) = 0$. Thus,

$$\begin{cases} \beta^+(t) = a\, u(t), \\ \beta^-(t) = 0, \end{cases} \tag{3}$$

for some $a > 0$. Substituting (3) into (2) results in the **logistic growth model**:

$$\begin{cases} u'(t) = \alpha\, u(t) \left(1 - \dfrac{u(t)}{K}\right) \\ u(0) = u_0 \end{cases} \tag{4}$$

where $\alpha := a\,U_0 > 0$, (measured in 1/day) represents the the average rate of Ukrainians crossing the border to Poland daily, and K, defined as $U_0 > 0$, is the *carrying capacity*, denoting the maximum number of Ukrainian evacuees that Poland can sustain per day. The parameter u_0 corresponds to the number of Ukrainians crossing the Polish border on February 24, 2022, set as $t_0 = 0$. The solution $u(t)$ to equation (4) represents the number of Ukrainian citizens entering Poland at time t. Problem (4) is autonomous and suitable for stability analysis.

3 Analysis of the Proposed Model and its Solution

3.1 Exact solution

As mentioned earlier, the logistic growth model (4) has an exact solution in *implicit form*, which can be obtained using the method of *separation of variables* [3]:

$$\frac{u(t)}{K - u(t)} = \frac{u_0}{K - u_0} e^{\alpha t}. \tag{5}$$

The solution (5) admits an explicit form:

$$u(t) = K - \frac{K}{\frac{u_0}{K - u_0} e^{\alpha t} + 1}. \tag{6}$$

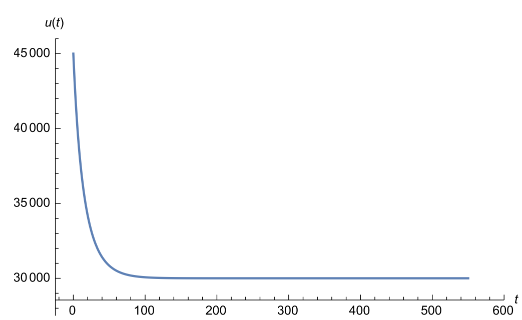


Figure 1: Solution of (4) with parameters $\alpha = 0.05$ and $K = 30{,}000$.

3.2 Stability analysis

Stability analysis in the context of autonomous ODEs is a mathematical technique used to determine the behavior of solutions near equilibrium points, see e.g. [3]. When one has an *equilibrium solution* for an autonomous ODE, it means that the system remains unchanged at that point, and the derivative of the solution with respect to time is zero, i.e., $u'(t) = 0$ at the equilibrium point. Stability analysis involves assessing how solutions behave when perturbed slightly from the equilibrium. If a system is *asymptotically stable* at an equilibrium point, a solution corresponding to any initial condition sufficiently close to that point will approach it as $t \to \infty$. In an *unstable equilibrium*, small perturbations will cause solutions to move away from the equilibrium point.

Specifically, the *equilibrium solutions* of (4) are

$$u(t) \equiv 0, \qquad u(t) \equiv K, \tag{7}$$

among which $u(t) = 0$ is *unstable* and $u(t) = K$ is *asymptotically stable* as $t \to \infty$.

The geometric representation of the stability analysis for (4), with parameters α and K as in Figure 1, is shown in Figure 2 for various values of u_0.

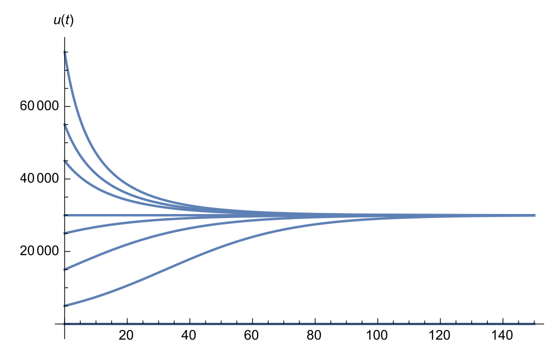


Figure 2: Qualitative study of equation (4): Plots indicate solutions of (4) with parameters as in Figure 1 corresponding to different initial conditions.

4 Parameter Estimation

To validate the model, next we consider a real data set, in particular, the observed data taken from the Polish Boarder Guard website and is displayed in Figure 3. The number of Ukrainian refugees crossing the border to Poland on February 24, 2022 according to this data is 31,260. This value will be used as the initial condition for our model below.

4.1 Least-square fit approach

In various fields such as science, engineering, economics, and more, mathematical models often have parameters that are not directly measured but are necessary to accurately describe the phenomenon being studied. **Pa-**

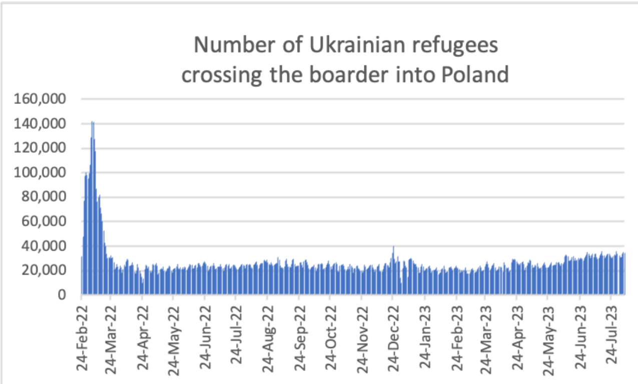


Figure 3: Recorded number of Ukrainian refugees crossing the Polish border at a given day.

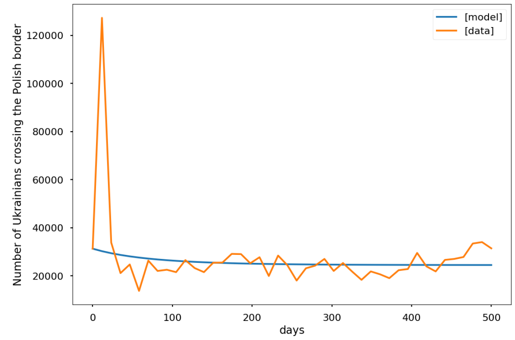


Figure 4: Downsampled observed data [red] based on the data given in Figure 3; and the solution to model (4) using fitted parameters (9) [blue].

rameter estimation aims to find the most likely values for these parameters based on the available data, see e.g. [2]. In the context of this project on Ukrainian migration, parameter estimation would involve determining the values of parameters in the proposed mathematical model (4) that describes the migration of Ukrainian evacuees to Poland during the war. This process helps align the model's predictions with the actual observed data, enhancing the model's accuracy and usefulness for understanding and predicting real-world events.

In the previous section, we used synthetic parameters in model (4). In this section, we aim to determine these parameters using the real data. However, this dataset is too dense to accurately determine just two parameters of the problem, α and K. To address this, we will *downsample the data*. Downsampling involves reducing the number of data points while attempting to preserve the essential characteristics of the original data, decreasing the dimensionality of the data, which helps regularize the ill-posed inverse problem and makes it more stable to solve. We are reducing 500 data points to 40 by applying uniform interval reduction, where the original data is divided into 40 equal-sized groups and each group is represented by a single data point.

Next, we outline the parameter estimation procedure, which relies on a *least-squares fit* [1]:

- We solve the model (4) using $u_0 = 31,260$ and parameters selected in Section 3.2 and the numerical procedures discussed earlier. We denote the obtained numerical solution as $g_{\text{par}}(t_i)$, hereafter referred to as *simulated data*. These simulated data points are defined for a set of time points $t_i \in [0, T]$, where $i = 1, \ldots, N$, with T chosen to be sufficiently large (e.g., $T = 550$) and measured in days.
- Let M be the number of *downsampled observed data* points, denoted as $\mathcal{G}_i$ for $i = 1, \ldots, M = 40$.
- We calculate the *residual* S as the sum of squares of the differences between the simulated and observed data:
$$S(\alpha, K) = \sum_{i=1}^{M} \left| g_{\text{par}}(t_i) - \mathcal{G}_i \right|^2, \tag{8}$$
- Finally, we minimize the function (8) with respect to the model parameters using a known optimization method for nonlinear functions, such as Newton's method (see e.g. [4]). For this project, we employed the *minimize* routine available in `Python`, where the code for numerical simulation was generated:
$$(\hat{\alpha}, \hat{K}) := \arg \min_{(\alpha, K)} S(\alpha, K).$$

In our simulations, the data $\mathcal{G}_i$ for $i = 1, \ldots, 40$ discussed in Section 4 were utilized. Therefore, we obtained the following

$$\hat{\alpha} = 1.13; \qquad \hat{K} = 24,447. \tag{9}$$

With these parameter values, we display the solution $u(t)$ to the model (4) in Figure 4.

Here, we remark that the goal in this project was to test two fundamental approaches to the inverse problems of identifying problem parameters (the *least square fit vs. PINNs*) based on the simplistic logistic growth model, and we did not aim to capture the "spike" of a large influx of Ukrainian refugees evacuating to Poland due to war, as it was an isolated incident that occurred on March 7, 2022. This rare event was not the main focus of our investigation. To accurately capture the data of the Polish Boarder Guard presented in Figure 3, one should either consider a different model than that in (4) or model the influx and outflux rates, $\beta^{\pm}$, as variable functions, which will be considered in a forthcoming paper.

4.2 Physics-informed neural networks

Physics-Informed Neural Networks (PINNs) are novel data-driven methodologies that combine the power of neural networks with the laws of physics. These networks are trained to understand and predict how things move and behave based on the fundamental rules of physics. PINNs set themselves apart from conventional neural networks by incorporating new components to the loss function, such as a residual of the differential equation and boundary and initial conditions [14].

Let $\lambda = (\alpha, K) \in \mathbb{R}^2$ be the vector of parameters related to the dynamics of the model (4). The training data has been discretized as $\{t_j, u_j\}_{i=0}^{N_{\text{data}}}$, where N_{data} is the amount of training samples. The approximation is defined as $\widehat{u}(t; \lambda, \theta)$ where θ is the vector of learnable parameters defined by the architecture of the neural network used, for example, for a fully connected neural network θ corresponds to the concatenation of biases and weights. In practice, during the training process, the elements of λ are also considered as learnable parameters using the loss function (10) $\mathcal{L}$ defined below, a linear combination of differential residuals, initial conditions and training data:

$$\mathcal{L}(\lambda, \theta) = \mathcal{L}_{\text{ode}}(\lambda, \theta) + \mathcal{L}_{\text{ic}}(\lambda, \theta) + \mathcal{L}_{\text{data}}(\lambda, \theta) \tag{10}$$

where $\mathcal{L}_{\text{ode}}$, $\mathcal{L}_{\text{ic}}$ and $\mathcal{L}_{\text{data}}$ are the loss functions of the system of differential equations, initial conditions and training data, respectively. While the overall loss function is decomposed in other three parts, the loss function of differential residuals is expressed as

$$\mathcal{L}_{\text{ode}} = \omega_{\text{ode}} \left[\widehat{u}' - \alpha \widehat{u} \left(1 - \frac{\widehat{u}}{K} \right) \right], \tag{11}$$

the loss function corresponding to the data may be expressed as:

$$\mathcal{L}_{\text{data}} = \frac{\omega_{\text{data}}}{N_{\text{data}}} \sum_{i=1}^{N_{\text{data}}} (u_i - \widehat{u}_i)^2,$$

and the loss function corresponding to the initial condition may be expressed just as

$$\mathcal{L}_{\text{ic}} = \omega_{\text{ic}} (u_0 - \widehat{u}_0)^2,$$

where ω_{ode}, ω_{data} and ω_{ic} are weights of the loss function of residuals, data and initial conditions, respectively. Note these are hyper-parameters of the framework. Algorithm 1 shows how to estimate $u(t; \lambda)$ and λ.

The computational application of Algorithm 1 to the same downsampled observed data mentioned in Section 4.1 was done using Python as programming language. In particular, the package `DeepXDE` [10], which allows flexibility for defining the differential equation residual, observed data, neural network architecture and automatic

Algorithm 1: PINNs algorithm.

Input : Training Data $\{t^j, u^j\}$.
Output: $\widehat{u}$ and $\widehat{\lambda}$.

Initialize $\widehat{\lambda}_0$ and $\widehat{\theta}_0$
Define time interval where the solution will be found.
Define loss function $\mathcal{L}(\widehat{\lambda}, \widehat{\theta})$, related to residual errors, initial conditions and training data.
Create a fully connected neural network with 1 neuron in the input layer and 6 neurons in the output layer (one per compartment).
Choose optimization hyper-parameters (e.g. Adam optimizer, learning rate and loss weights).
for $\mathtt{iter} = 1, \ldots, \mathtt{max\_iter}$ **do**
    Compute total loss $\mathcal{L}(\widehat{\lambda}_{\mathtt{iter}-1}, \widehat{\theta}_{\mathtt{iter}-1})$ using auto-differentiation for ODE residuals.
    Train neural network with optimizer algorithm and update $\widehat{\theta}_{\mathtt{iter}-1}$ to $\widehat{\theta}_{\mathtt{iter}}$.
    Get approximation $\widehat{u}_{\mathtt{iter}}$ and $\widehat{\lambda}_{\mathtt{iter}}$.
end
Return $\widehat{u}_{\mathtt{max\_iter}}$ and $\widehat{\lambda}_{\mathtt{max\_iter}}$.

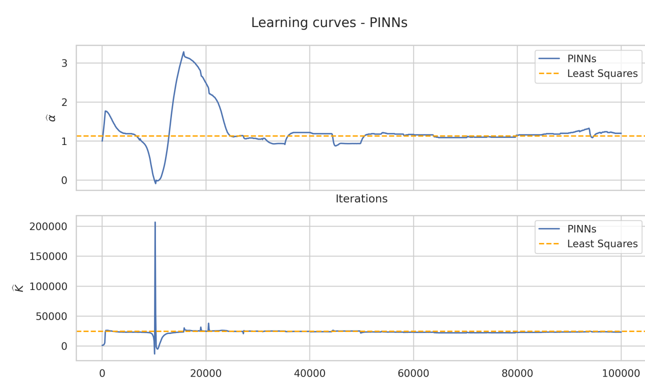
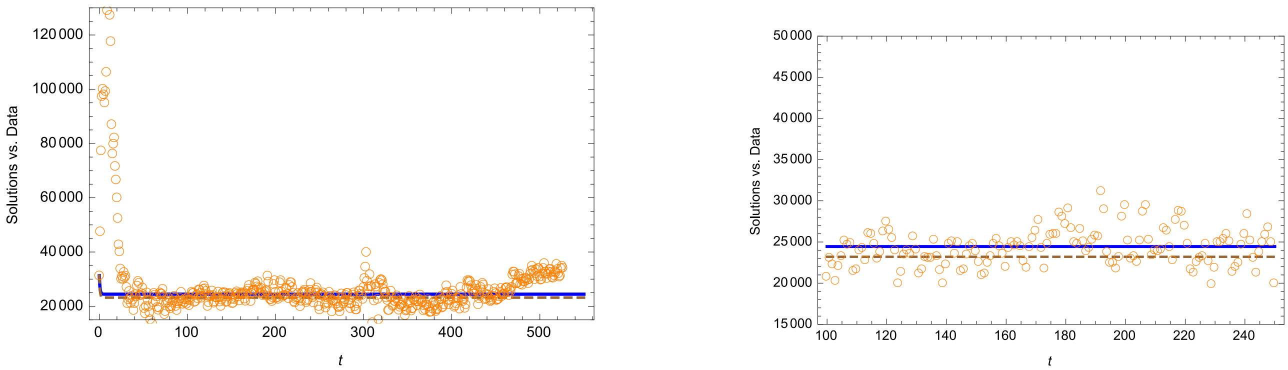


Figure 5: Parameter estimation learning curves using PINNs.



(a) (b)

Figure 6: Comparison of two solutions to (4) obtained by the least square fitting (solid thick line −, blue) and the solution to (4) obtained by the PINNs (dashed line −−, purple) with observed data of Figure 3 (circles ∘, orange): (a) Full range of observed data $t \in [1, 550]$; (b) Zoom-in of the same results vs. data on the interval $t \in [100, 250]$.

differentiation backend. For this experiment, we used a fully connected neural network of 3 hidden layers with 64 neurons per layer and ReLu as activation function. The optimizer selected was Adam with a learning rate of 0.01 and 100,000 training iterations. Finally, the weight hyperparameters were $\omega_{\text{ode}} = 1/5$, $\omega_{\text{data}} = 4/5$ and $\omega_{\text{ic}} = 0$. This particular selection of weights was based on experimentation. For example, since the observed data already included the initial condition, it was not necessary to include the initial condition residual. On the other hand, in order to fit the model faster, the weight of the data residual is four times bigger than the differential equation loss. This methodology led to the following estimations:

$$\hat{\alpha} = 1.199; \qquad \hat{K} = 23,201. \tag{12}$$

These estimates are reasonably close to what was obtained by the Least-Squares methods that helps to validate our estimation. Figure 5 shows the learning curves of this approach compared with the estimated values from the least squares methodology, the horizontal axis corresponds to the iteration and the vertical axis to the current approximation the parameter.

The solutions to the discussed model (4) with parameters α and K obtained by the least square method (9) and by PINNs (12), respectively, are displayed in Figure 6. Comparing the least squares and PINNs approaches for parameter estimation in ODE models allows validating the results, assessing robustness, gaining insights, and guiding further methodological developments. The current investigation, which resulted in the development of numerical schemes and codes for both solving the ODE and training neural networks to solve the ODEs while fitting the model to data, has set the groundwork for future studies on more complicated migration models.

5 Discussion and Conclusions

The current investigation aimed at the study of the number of Ukrainian refugees evacuating to Poland due to war with Russia started on February 24, 2022. We employed a social dynamics model with specific parameter values, resulting in the logistic growth model. The *logistic growth model* provides a baseline understanding of the fundamental dynamics at play, allowing one to establish a foundation upon which to build more sophisticated models. It offers insights into the intrinsic growth rate α and carrying capacity K that govern the migration process, which are crucial parameters to consider even in complex scenarios. Starting with a simple model helps identify the most influential factors driving migration and sets the stage for study of a more complicated model that will incorporate additional complexities, such as gender or spatial distribution, or policy regulations/interventions, and others, in a systematic manner. Aforementioned factors will be accounted in forthcoming studies.

Additionally, our study encompassed analysis of this problem, including stability analysis of equilibrium solutions. Furthermore, we utilized parameter estimation techniques to discern realistic parameter values.

It may be noted that one can enhance the model presented here by incorporating a time-dependant migration rate and comparing fitting approaches. While this enhancement may lead to a more complex model that may provide additional insights, the simpler logistic growth model presented herein, provides a solid foundation for understanding the overall migration dynamics. Understanding the limitation of the simpler model also would allow one to refine or extend to more complex models in future studies.

Finally, in Section 4, we estimated the model parameters through the *least square* fit procedure and an alternative way for determining the model's parameters using a neural networks approach called PINNs [15]. The two methods are in agreement, showing the robustness of the numerical parameter estimation techniques used. This also suggests that the *daily* migration of Ukrainian refugees to Poland due to the war is expected to eventually stabilize at the level of approximately 24 thousand people. Also, note that comparing the least squares and PINNs methods (see Figure 6) for the simple logistic model presented has provided a valuable reference point to identify the strengths and limitations of each approach.

Lastly, it is crucial to note that the data presented in Figure 3 is reported on a daily basis. A straightforward summation of these figures projects an improbable scenario of over 14 million people migrating from Ukraine to Poland between February 24, 2022, and the end of July 2023. This discrepancy arises because the model (2), under assumption (3), does not account for Ukrainians returning to Ukraine (i.e., $\beta^- = 0$) or individuals crossing the border repeatedly, leading to the counting of the same individuals multiple times. To provide more realistic projections, a refined model that considers these factors is needed, and, as mentioned above, will be reported in forthcoming papers.

Author Contributions

The first author, D. Sitalo, was responsible for developing the mathematical model (4) presented in Section 2, stability analysis discussed in Section 3.2, parameter estimation by least-square fit introduced in Section 4.1, and for typing up the current manuscript. The second author, A. Ogueda-Oliva, was responsible for validating the numerical results by finding values of parameters using PINNs, explored in Section 4.2. The third author, P. Seshaiyer, provided mentorship, guidance, and support to both first and second authors as well as assisted in typing, proof-reading and funding, that are gratefully acknowledged. Also, the last author acknowledges partial support for this work from the National Science Foundation under grant DMS-2230117 and DMS-2232739.

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