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The Reemergence of Eradicated Disease due to Ecological Impact of Climate Change

Claudia Kolakowski
Illinois State University, ckolako@ilstu.edu

THE REEMERGENCE OF ERADICATED DISEASE
DUE TO ECOLOGICAL IMPACT OF
CLIMATE CHANGE

CLAUDIA KOLAKOWSKI

29 Pages

Global warming is radically changing aspects of the Earth. As scientists continue to research the effects, the ramifications of melting permafrost is coming to light. We build off of a previously existing Anthrax model in the hopes to include climate change as a factor in Anthrax spread. Chapter II develops a simplified version of an Anthrax model. Parameters for the model are found by using previous research and eigenvalues are analyzed in order to find thresholds and equilibria. Chapter III consider the general solutions of the model through eigenvalues and eigenvectors. This model is then extended to include a parameter that signifies the melting of permafrost to reveal frozen carcasses in Chapter IV. As this parameter increases, the resulting number of anthrax cases which lead to death increases. The final chapter extends the thawing period of permafrost by starting the melting point in the year at an earlier date. Data has shown that in previous decades, the active layer of permafrost began to melt around June 1st. As global temperatures increase, the permafrost is beginning to melt earlier. Our figures display the change this has on Anthrax spread. The goal is to understand how severe the effects of climate change could be on the reemergence of Anthrax.

KEYWORDS: Anthrax, Permafrost, Global Warming, Disease Modeling

THE REEMERGENCE OF ERADICATED DISEASE DUE TO ECOLOGICAL IMPACT OF CLIMATE CHANGE

CLAUDIA KOLAKOWSKI

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CLAUDIA KOLAKOWSKI

COMMITTEE MEMBERS:

Olcay Akman, Chair

Joanna Wares

Sara Krehbiel

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CHAPTER I: INTRODUCTION

In the 1800's, scientists were performing experiments which suggested that human-produced carbon dioxide and other gases could insulate the Earth [14]. By the 1980's, there was a sharp increase in global temperatures[14], and the idea of global warming became a serious cause of concern. Along with continued research on what is causing this global warming, scientists began to dig into the possible ramifications of a warming climate. A recently considered effect is the melting of permafrost. Permafrost is permanently frozen ground which remains at or below 0^o C for at least two years[22]. It keeps microbes, carbon, poisonous mercury and soil locked in. Bacteria and viruses are preserved for hundreds of thousands of years, and all they need is for the permafrost to thaw in order to revive.

There has been a recent Anthrax outbreak in Siberia, where no cases of Anthrax were reported in over seventy years[17]. It was discovered that decades ago, anthrax infected reindeer were buried in the permafrost, perhaps because people did not want to waste firewood to burn the carcasses[17]. With recent rises in temperature, the ground where these carcasses were buried began to melt. Water ran through the burial grounds, picking up newly defrosted Anthrax bacteria and bringing these microbes to rivers and streams that cattle drink out of.

Anthrax is an infectious disease caused by bacteria known as *Bacillus anthracis.* Wild animals become infected with it when they breathe in or ingest spores in contaminated soil, plants or water. When an animal dies of anthrax, the bacteria is present in most tissues of the body. Sporulation occurs when the bacterium comes into contact with free oxygen in the air as exudates are released from a dead or dying animal. If the tissues are not exposed to oxygen, the bacteria cannot form the spores that infect other animals and quickly die off. Scavengers open the carcass and expose the spores to oxygen, which causes infectious spores to begin to form. Spores continue to live in the soil long after the animal has decomposed. They end up deep in the soil and are brought to the surface when the ground is disturbed (rain, erosion, etc.). Anthrax spores require alkaline conditions to survive and are rarely seen in the winter since their optimal temperature for germination is around 20^o C. Spores are difficult to destroy and can survive for decades, causing Anthrax to be a continuous problem among herd animals.

Hahn and Furniss[12] proposed a series of differential equations to model the cycle of anthrax. Since then, other researchers have proposed more complex models with various different attributes, such as Sinkie et al. [25] who included and analyzed a model with an added infected compartment. Friedman and Yakubu [10] also proposed a model to explain the spread of anthrax which included carcass induced environmental contamination. Of the previously proposed models, none included climate change and melting permafrost in their models. With this study, we propose a new model which now includes climate change. In Chapter II we propose the new model and find the equilibrium solutions. In Chapter III we introduce seasonality into our model by adding a new term, and the change in this term will be the change in seasonality due to climate change. We then extend the season in which anthrax is active and germinating, which causes new infections, in Chapter IV. These alterations will symbolize the change in climate and hopefully help us understand how global warming can effect the spread of anthrax.

CHAPTER II: MODEL DEVELOPMENT

We begin with the Hahn and Furniss model where climate change has no impact, and develop a new model in order to simplify assumptions. The Hahn and Furniss model is given by:

$$\frac{dv}{dt} = -av \tag{1}$$
$$\frac{da}{dt} = -\alpha a + \beta c \tag{2}$$
$$\frac{dc}{dt} = av - \delta c \tag{3}$$

In the equations II.1, II.2 and II.3, $v(t)$ is the number of vulnerable animals at time t, $c(t)$ is the number of carcasses (where the animals died of anthrax), and $a(t)$ is the environmental contamination. The parameters are α, which is the death of spores or their removal from the environment, β, which is the rate at which contamination is disseminated from carcasses, and δ, which is the decay-rate of carcasses.

For the new model, we will assume a constant vulnerable population causing $\frac{dv}{dt} = 0$, which leaves us with only two equations. Since we no longer have v as a term, there needs to be a new constant to symbolize the vulnerable animals getting infected with anthrax, dying, and then becoming carcasses. Let this variable be k, where k is the rate at which vulnerable animals are coming into contact with an anthrax contaminated environment and dying. Comparing to the Hahn and Furniss model, this is similar to the $v * a$ term. This gives us the following model:

$$\frac{da}{dt} = -\alpha a + \beta c \tag{4}$$
$$\frac{dc}{dt} = ka - \delta c \tag{5}$$

This model can be read more easily with a compartmental model, which is seen in **Figure 1**.

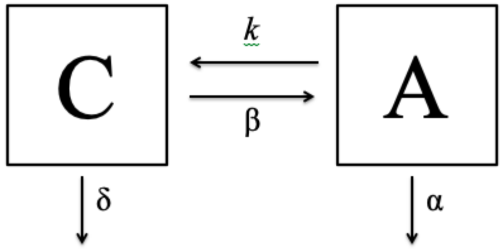

Figure 1: Compartmental Model

In **Figure 1**, the compartment labeled with a C is the amount of carcasses and the compartment labeled with an A is the amount of anthrax spores in the environment. The vulnerable animals get infected from the contaminated environment at a rate of k, so this is the incoming population to the carcass compartment. These carcasses decay at a rate of δ, which is seen with the arrow leaving the carcass compartment. The carcasses are also scavenged and release spores that contribute to the contaminated environment at a rate of β. Then we have spores decaying at a rate of α, which is seen with the arrow leaving the anthrax spore compartment.

Before adding an environmental component, we use equations II.1, II.2 and II.3 to obtain parameter estimates. To estimate the parameter of the decay rate of carcasses, δ, Pantha et al. [21] used experimental data by Bellan et al [4] where soil samples from an infected carcass site were collected over 365 days and the number of spores per gram of soil were counted. Two differential equations were developed to describe the rate of change of spores and the carcasses. From these ordinary differential equations along with the experimental data, an estimate of carcass decay rate, $\delta = .2816$, was found. In order to estimate β, α, or k, we first need to know how many spores it would take for one animal to get infected. Schlingman et al. [3] found that oral administration of 150 million spores proved fatal to most cattle. So the unit for the three parameters involving spores will be 150 million spores. For the β estimate, we will use a value of .03 which is found to be the rate at which spores are released from a partially scavenged carcass [4]. The decay rate of anthrax spores was found to be .026 ± .007 in topsoil [30]. We use $\alpha = .026$ for our model. Lastly, as an estimate for our new parameter, k, we use a range of values to show how many new animal deaths per 150 million spores we have per day. **Figure 2** depicts the outputs of the model in equations II.4 and II.5.

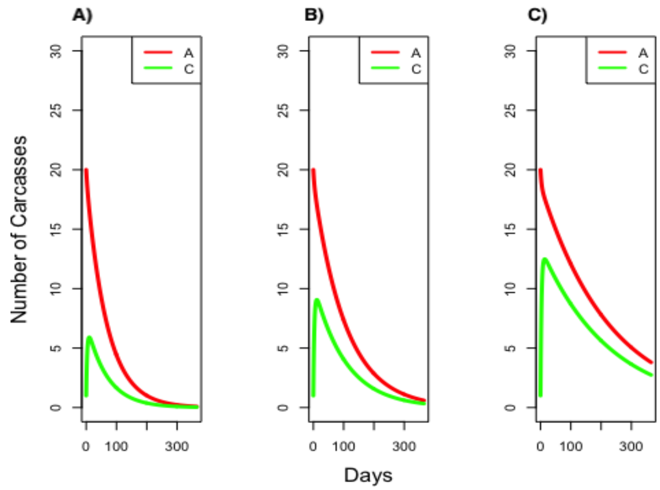

Figure 2: Model run with initial values $A(0) = 20$ and $C(0) = 1$. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$ and $k = .1, .15, .2$ for graphs **A, B, C** respectively

Since we are not using data, the initial values are arbitrary and remain the same for all three graphs. The amount of anthrax spores in the environment, which is represented by the red line, starts at 20 units. This was earlier defined as 150 million spores per unit. This red line declines quickly in graph **A** and has a slower decline as our k parameter increases, seen in graphs **B** and **C**. The number of carcasses starts out at one and in graph **A** we see that the highest amount of carcasses we reach is approximately 6. With a larger rate of susceptible animals coming into contact with the contaminated environment, we see that this peak rises. In graph **B**, our peak reaches almost 10 carcasses and in graph **C**, the peak is at around 12 carcasses, which is double the amount from graph **A**. It is also seen that the time it takes until the spores in the environment as well as the non-decayed carcasses to reach zero is extended. In graph **A**, a zero equilibrium is reached at around day 200, whereas in graph **B** this is extended to around day 300 and in graph **C**, the populations don't reach zero until way past day 300.

Stability of Equilibrium

In order to find the equilibrium points of our model we set our differential equations to zero, since an equilibrium point is one where our system will not change over time, and solve. We leave the parameter k as a variable:

$$0 = -.026a + .03c \tag{6}$$

$$0 = ka - .2816c \tag{7}$$

From equation II.6 we get $\frac{.026}{.03}a = c$ and equation II.7 is simplified to $.2816c = ka$. Substituting our value of c from the first equation into the second equation we get

$$\begin{aligned} .2816\left(\frac{.026}{.03}\right) &= ka \\ .244a &= ka \\ 0 &= -.244a + ka \\ 0 &= (-.244 + k)a \end{aligned}$$

So we see that we will have an equilibrium at $a = 0$. Plugging $a = 0$ into equation **II.6**, we find that $c = 0$. Thus we have an equilibrium at $(a, c) = (0, 0)$, which is the origin. We now determine the stability of the origin by looking at the eigenvalues obtained from our differential equations. We obtain the matrix:

$$A = \begin{pmatrix} -\alpha & \beta \\ k & -\delta \end{pmatrix}$$

In order to find the eigenvalues, we set $det(A - \lambda I) = 0$ and solve for λ as follows:

$$A - \lambda I = \begin{pmatrix} -\alpha - \lambda & \beta \\ k & -\delta - \lambda \end{pmatrix}$$

$$\begin{aligned} det(A - \lambda I) &= (-\alpha - \lambda)(-\delta - \lambda) - \beta k \\ &= \lambda^2 + (\alpha + \delta)\lambda + (\alpha\delta - \beta k) \end{aligned}$$

Using the quadradic formula, we can solve for $0 = \lambda^2 + (\alpha + \delta)\lambda + (\alpha\delta - \beta k)$:

$$\lambda = \frac{-(\alpha + \delta) \pm \sqrt{(\alpha + \delta)^2 - 4(\alpha\delta - \beta k)}}{2}$$

In order to interpret these resulting eigenvalues, we can look at two different cases.

Case 1: $\underline{\alpha\delta < \beta k}$

Here we can see that the term $(\alpha\delta - \beta k)$ will be a negative value, and when multiplying that value by negative 4, we see there will always have a positive number under square root. This means our eigenvalues will be real numbers. The origin has different stability based on whether there are two positive real eigenvalues, two negative real eigenvalues, or one of each. Let us determine the signs of our eigenvalues in Case 1. Since the denominator is a positive constant, the numerator will determine the sign of the eigenvalue. For $\lambda_1 = -(\alpha + \delta) - \sqrt{(\alpha + \delta)^2 - 4(\alpha\delta - \beta k)}$, we will clearly have some negative value since we know the square root of those terms will be a positive real number. For $\lambda_2 = -(\alpha + \delta) + \sqrt{(\alpha + \delta)^2 - 4(\alpha\delta - \beta k)}$, we get a positive eigenvalue since our condition in this case is that $\alpha\delta < \beta k$. Therefore in Case 1, we have one positive and one negative distinct real eigenvalues, making our origin a saddle point.

Case 2: $\underline{\alpha\delta > \beta k}$

Now the $(\alpha\delta - \beta k)$ term will be positive, so we will have a positive number multiplied by negative 4. This leads to the possibility of having a negative number under the radical, which would give us complex eigenvalues.

a) In order for the value under the radical to be negative, we would need to have $(\alpha+\delta)^2 < 4(\alpha\delta - \beta k)$. With some algebra we can rearrange this as follows:

$$
\begin{aligned}
(\alpha+\delta)^2 &< 4(\alpha\delta - \beta k) \\
\alpha^2 + 2\alpha\delta + \delta^2 &< 4\alpha\delta - 4\beta k \\
\alpha^2 + \delta^2 &< 2\alpha\delta - 4\beta k
\end{aligned}
$$

This would only be possible if the parameter k was to have a negative value. This will never happen in our model, so we know that we will not have complex eigenvalues.

b) Since $(\alpha+\delta)^2 < 4(\alpha\delta - \beta k)$ does not hold true for our model, it must be true that $(\alpha+\delta)^2 > 4(\alpha\delta - \beta k)$. This would result in real eigenvalues, as in Case 1. Similarly to before, we see that $\lambda_1 = -(\alpha+\delta) - \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta k)}$ will result in a negative eigenvalue. However, for $\lambda_2 = -(\alpha+\delta) + \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta k)}$, we now have the condition that $\alpha\delta > \beta k$ making this eigenvalue negative. Thus in Case 2, we have two real distinct negative eigenvalues, which gives us an asymptotically stable origin.

We would like the origin to be an asymptotically stable equilibrium point, so we will choose a parameter value for k to satisfy $\alpha\delta > \beta k$. By plugging in the parameter values $\alpha = .026, \delta$=.2816 and β=.03 found earlier in this chapter, we see that we need $k < .244$ in order to meet this condition. This means that if we use a k-value which is less than .244, our populations of carcasses and spores in the environment will eventually reach the asymptotically stable equilibrium at the origin. In the next chapter we will solve for the general solution of our model.

CHAPTER III: GENERAL SOLUTIONS

Using the parameters $\alpha = .026, \delta$=.2816, β=.03 and $k = .2$, we will solve for the general solution. We start by finding eigenvalues and eigenvectors. To solve for the eigenvalues, we plug our parameters into the solution found earlier:

$$\lambda = \frac{-(\alpha+\delta) \pm \sqrt{(\alpha+\delta)^2 - 4(\alpha\delta - \beta k)}}{2}$$

From this we get $\lambda_1 = -.00438$ and $\lambda_2 = -.303242$. We will use these eigenvalues to solve for our eigenvectors. We want to find where $(A - \lambda I) \cdot \vec{V} = 0$, where I is the identity matrix and A is our parameter matrix, by inputting our distinct eigenvalues. Starting with λ_1 we have:

$$(A - \lambda_1 I)\vec{V} = \begin{pmatrix} -.026 - (-.00438) & .03 \\ .2 & -.2816 - (-.00438) \end{pmatrix} \cdot \begin{pmatrix} a \\ c \end{pmatrix}$$

This gives us the our first eigenvector:

$$V_1 = \begin{pmatrix} 1 \\ .721 \end{pmatrix}$$

Using the second eigenvalue in the same manner we get:

$$(A - \lambda_2 I)\vec{V} = \begin{pmatrix} -.026 - (-.303242) & .03 \\ .2 & -.2816 - (-.303242) \end{pmatrix} \cdot \begin{pmatrix} a \\ c \end{pmatrix}$$

This yields our second eigenvector:

$$V_2 = \begin{pmatrix} -.10821 \\ 1 \end{pmatrix}$$

We can now use the eigenvalues and eigenvectors to come up with a general solution. This general solution can be used to find the expected outputs from our populations at any time t once initial values from a set of data are obtained. The form of a general solution is as follows:

$$\begin{bmatrix} a(t) \\ c(t) \end{bmatrix} = C_1 V_1 e^{\lambda_1 t} + C_2 V_2 e^{\lambda_2 t}$$

Plugging in the eigenvalues and eigenvectors that we solved for, we get:

$$\begin{bmatrix} a(t) \\ c(t) \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ .721 \end{bmatrix} e^{-.00438t} + C_2 \begin{bmatrix} -.10821 \\ 1 \end{bmatrix} e^{-.303242t}$$

To find C_1 and C_2 we would need initial conditions from a set of data, however, we want the general solution to be applicable to any set of data. Let us look at the overall solutions at $t = 0$.

$$\begin{bmatrix} a(0) \\ c(0) \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ .721 \end{bmatrix} + C_2 \begin{bmatrix} -.10821 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a(0) \\ c(0) \end{bmatrix} = \begin{bmatrix} C_1 - .10821C_2 \\ .721C_1 + C_2 \end{bmatrix}$$

Now that we have developed our model and found, as well as analyzed, the parameter estimates, we will add seasonality into the differential equations. In the next chapter, a new parameter will be introduced in order to include seasonality, and the change of this parameter will signify the change of seasonality due to climate change.

CHAPTER IV: ADDING CLIMATE FACTOR

In this chapter we will add seasonality to our model. Research shows that permafrost in areas of Siberia is thawing earlier and to a greater depth. When permafrost melts to deeper levels, there is the possibility that more carcasses will become exposed and scavenged, leading to the spread of new anthrax cases. We wanted to derive a model that interpreted what could happen to the spread of anthrax when permafrost melted to greater depths. To do this, we first needed a baseline that includes seasonal trends that we could use to compare. Data from the 1980's was used to model when the active layer of permafrost would melt seasonally [20]. In June the active layer begins to thaw and then in October the active layer is seen to freeze from the top downwards, limiting the spread of anthrax since the contaminated carcasses would be frozen. This means that from the 152^{nd} to the 303^{rd} day of the year, anthrax would be most likely to spread. We added a new parameter, $F(t)$, into our model. This parameter will signify the melting of permafrost to reveal frozen carcasses. This will be a time-dependent piece-wise function, where

$$f(t) = \begin{cases} \gamma & t \in \text{June } 1^{st}\text{-October } 1^{st} \\ 0 & \text{if else} \end{cases}$$

Our function $f(t)$ captures the effect of permafrost melting in the summer months. It is parameterized by γ, where γ will only be active during the period of time where permafrost is thawing, thus additional carcasses will only be exposed and at risk of spreading anthrax spores during this season. Our new model is:

$$\frac{da}{dt} = -\alpha a + \beta c$$
$$\frac{dc}{dt} = ka - \delta c + f(t)$$

When we are at $t < 151$ or $t > 304$, our model runs as it did in Chapter II. This means that if there are infected carcasses in the area, the disease will spread and peak, but then decline towards the zero equilibrium. However, at time $151 < t < 304$, we have additional carcasses being added to the carcass compartment at a rate of γ. These carcasses are ones that have been frozen over in the previous year, along with the anthrax spores running through their body, and are now exposed due to the melting of the active layer of permafrost. This is now a nonhomogeneous system, and so we find a new set of equations for the general solution.

Solution to Nonhomogeneous System

The following complementary solution is shown in Chapter III:

$$x(t) = C_1 \begin{bmatrix} 1 \\ .721 \end{bmatrix} e^{-.00438t} + C_2 \begin{bmatrix} -.10821 \\ 1 \end{bmatrix} e^{-.303242t}$$

From this we get a matrix from which we find the particular solution with added γ.

$$X = \begin{bmatrix} e^{-.00438t} & -.10821e^{-.303242t} \\ .721e^{-.00438t} & e^{-.303242t} \end{bmatrix}$$

The particular solution can be found using the formula $x_p = X \int X^{-1} \overrightarrow{g}\, dt$. The inverse of matrix X is found to be:

$$X^{-1} = \begin{bmatrix} .9276e^{.00438t} & .1004e^{.00438t} \\ -.6688e^{.303242t} & .9276e^{.303242t} \end{bmatrix}$$

Multiplying this by the γ vector $\begin{bmatrix} 0 \\ \gamma \end{bmatrix}$, where zero represents no defrosted carcasses and γ is the parameter representing the added frozen carcasses from previous years, we have:

$$X^{-1}\overrightarrow{\gamma} = \begin{bmatrix} .9276e^{.00438t} & .1004e^{.00438t} \\ -.6688e^{.303242t} & .9276e^{.303242t} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \gamma \end{bmatrix} = \begin{bmatrix} .1004\gamma e^{.00438t} \\ .9276\gamma e^{.303242t} \end{bmatrix}$$

We take the integral of this:

$$\int X^{-1}\overrightarrow{\gamma}\,dt = \begin{bmatrix} 22.92\gamma e^{.00438t} \\ 3.06\gamma e^{.303242t} \end{bmatrix}$$

Now for the particular solution, we multiply this integral by the original matrix, X:

$$\begin{aligned} x_p = X\int X^{-1}\overrightarrow{g}\,dt &= \begin{bmatrix} e^{-.00438t} & -.10821e^{-.303242t} \\ .721e^{-.00438t} & e^{-.303242t} \end{bmatrix} \cdot \begin{bmatrix} 22.92\gamma e^{.00438t} \\ 3.06\gamma e^{.303242t} \end{bmatrix} \\ &= \begin{bmatrix} 22.92\gamma - 2.4801732\gamma e^{-.298862t} \\ 3.06\gamma + 2.20626\gamma e^{.298862t} \end{bmatrix} \end{aligned}$$

We combine our particular solution and complementary solution in order to get a general solution:

$$x(t) = C_1 \begin{bmatrix} 1 \\ .721 \end{bmatrix} e^{-.00438t} + C_2 \begin{bmatrix} -.10821 \\ 1 \end{bmatrix} e^{-.303242t} + \begin{bmatrix} 22.92\gamma - 2.4801732\gamma e^{-.298862t} \\ 3.06\gamma + 2.20626\gamma e^{.298862t} \end{bmatrix}$$

As previously mentioned, we want to begin with a model where climate change is not a factor. Starting with low initial conditions will quickly lead our populations to the zero equilibrium and a spike is only seen during the summer months where the only anthrax that is spread would be that of the frozen carcasses that were recently exposed. Increasing the recently added gamma parameter will increase the amount of frozen carcasses thawed and added to the model, thus causing a greater spike in cases in the summer months, and potentially having more infected carcasses frozen during the winter months which would lead to a greater number of cases the following year. Though our populations are correlated, there are many possibilities for the initial conditions for an accurate real life simulation. Anthrax spores release into the ground surrounding infected carcasses and through seasonal changes and ground disturbances, these spores may make it to the surface more often and at a faster rate than the entire carcass itself. This would leave us with a higher contaminated environment at the start, A(0). It could also be the case that the frozen carcasses resurface, but have not yet been scavenged. If the anthrax spores have not come into contact with oxygen, they will not be active and will remain in the infected hosts body. This would give us a higher initial carcass value at the start, C(0).

We did not work with real data, thus our initial values will be arbitrary. We set A(0)=15 and C(0)=10. We will also set our new parameter, γ, equal to some arbitrary value. There is still a lot of research to be done and data to be collected on this topic, so we will simply be looking at a simulation of what could happen and then comparing this outcome with more extreme situations once global climate change is introduced. We start with $\gamma = .5$, which could be interpreted as one additional carcass is exposed from the permafrost every other day during the summer months. Using these values, we see our new model depicted in **Figure 3**.

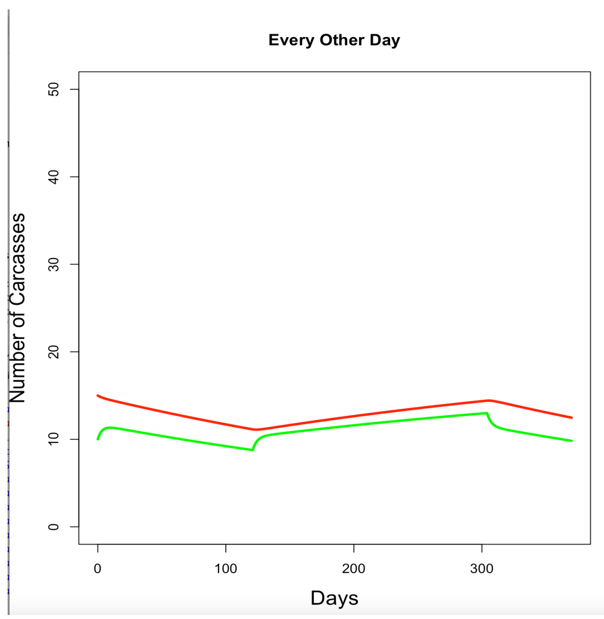

Figure 3: Model runs with initial conditions A(0)=15 and C(0)=10. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$. and $\gamma = .5$. Red Line=Spore Population, Green Line=Carcass Population

In **Figure 3**, we see from time zero to time 151 the model runs without the additional parameter, allowing the disease to have its initial spike but then steadily decrease towards the asymptotically stable zero equilibrium, as seen earlier in **Figure 2** before the new parameter was added. Then, once the active layer thaws on day 152 of the year, the previously frozen carcasses become exposed and cause the Anthrax spores to reemerge. After the summer months on day 304 (October 1st), our system returns to normal and the populations continue towards their zero equilibrium. At the end of the year in this simulation, the populations almost return back to their initial values. We take note of this in order to be able to compare this result with other graphs once the parameters change. Our carcass populations ends at about 9.94 units and our anthrax spore population ends at about 12.62 units. We will now compare this to the outcomes of the populations once γ is increased. **Figure 4** displays the graphs with the changed parameter.

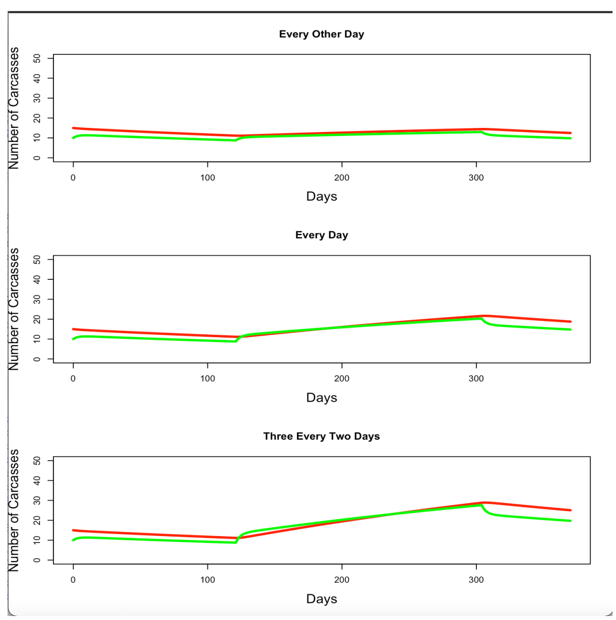

Figure 4: Model runs with initial conditions A(0)=15 and C(0)=10. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$. and $\gamma = .5, 1, 1.5$ for graphs "Every Other Day," "Every Day," and "Three Every Two Days." Red Line=Spore Population, Green Line=Carcass Population

In **Figure 4**, graphs **Every Day** and **Two Every Three Days** have the same initial values as graph **Every Other Day**, but we increased the γ parameter. That means that the permafrost melted to a greater depth, resulting in more carcasses exposed. For the graph titled "Every Day," our $\gamma = 1$, which means that an additional carcasses is added to the population from the melted permafrost every day, as opposed to every other day when $\gamma = .5$. We see that by doubling the amount of exposed carcasses, the ending value of both populations is much greater. The carcass population ends at about 14.96 units comparing to 9.94 units when $\gamma = .5$, and the anthrax population finishes the year at about 19 units, which is much higher than the 12.84 units seen before. Now looking at the graph when the γ parameter is tripled, "Two Every Three Days" has γ set to 1.5, which would result in two carcasses being exposed from the permafrost every three days. Again, we see that both of our populations have increased at the end of the year. Our carcass population jumps to 19.98 and our anthrax spore populations goes to 25.36. The increase over just one year is very clear. Now let us take a look at what would happen over four years.

We will now look at the continuous cyclical trend over four years, with the amount of carcasses at the end of each thawing period becoming the starting point for the following year. **Figure 5** shows us that since there are more carcasses being frozen at the end of the summer season, there are more anthrax spores being contained in this frost and surviving the upcoming winter season. These carcasses are exposed at the beginning of the next thawing season, leading to a more abundant anthrax spore environment. This leaves more animals vulnerable to obtaining the disease. At the end of four years with $\gamma = .5$, seen in graph "Every Other Day," our anthrax cases are even declining. If there is no introduction of a new contaminated population to the original population, the disease will continue decreasing towards its zero equilibrium, even with the minor spike in cases over the summer months. All of the graphs started with the initial condition of $C(0) = 10$. In graph "Every Other Day" where $\gamma = .5$, the population of carcasses at the end of the four years is C(1460)= 7.65 (green line). We compare this to the second graph "Every Day" where C(1460)=14.93, and in the third graph "Two Every Three Days" we have C(1460)=22.22. We have shown that increasing this parameter of seasonality leads to the result of a substantial increase in anthrax cases. With the introduction of climate change causing more and more carcasses to be exposed in the summer months, the decreasing trend changed into an increasing trend. In the next chapter, we will look into starting the thawing period of the active layer of permafrost at an earlier time. It has been stated that researchers noticed permafrost melting earlier in the season than previously and we would like to model the effect this would have on the spread of anthrax.

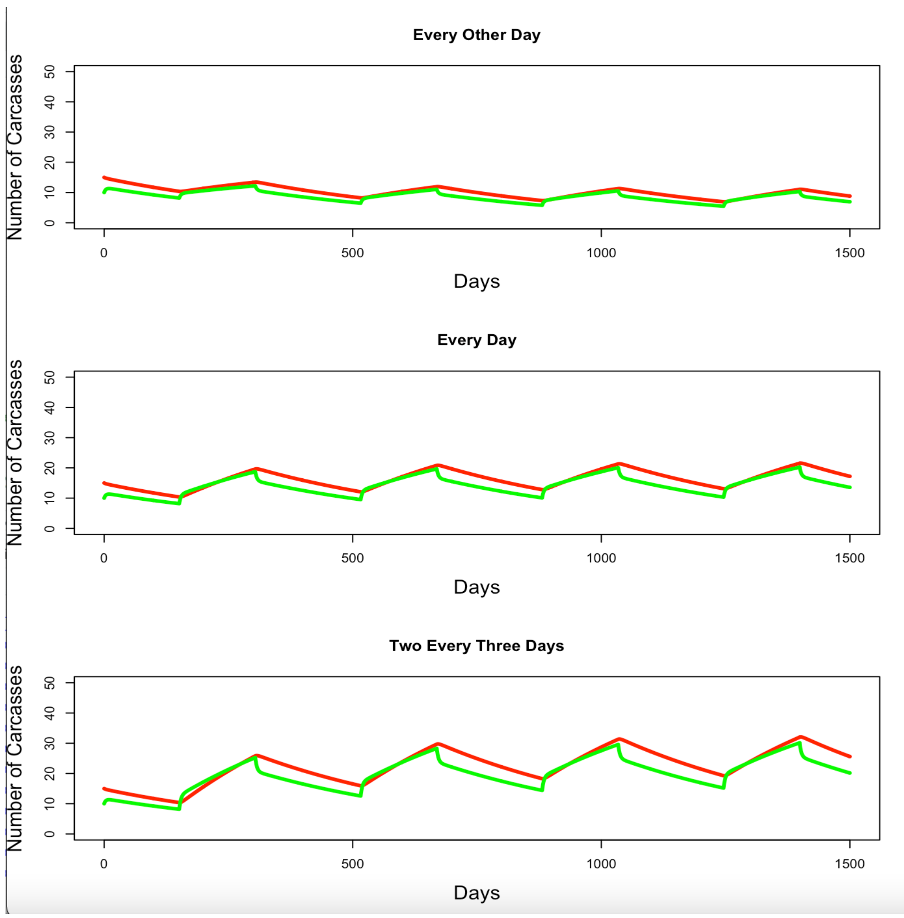

Figure 5: Our time dependent model over four years with initial conditions A(0)=15 and C(0)=10. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$ and $k = .2$. $\gamma = .5, 1, 1.5$ for graphs "Every Other Day," "Every Day," and "Two Every Three Days" respectively. Red Line=Spore Population, Green Line=Carcass Population

CHAPTER V: EFFECT OF CLIMATE CHANGE

We saw that increasing the intensity of our new parameter γ, which refers to a change in seasonality due to climate change, yielded higher anthrax related deaths. We would now like to model what would happen if the season was extended, since permafrost is melting earlier due to global climate change. We take a look at a few different scenarios by extending the start of thawing of the active layer by different sets of time. We will keep our parameters the same and set $\gamma = .5$ in order to have the start of the thawing period be the only variable. In **Figure A-2**, we see that in one years time, the change in the initial start of thawing does not make a very huge difference if the thawing period started two weeks or one month earlier. The number of cattle carcasses increases very slightly with a longer thawed season. This is very different than what we saw in **Figure 4** where even in one years time, the increase in exposed carcasses made a visible difference. Now let's see what the change would be when continuing this extended thawing period over the span of four years.

In **Figure 6** we see the output of only the carcasses from the model and not the anthrax spores in the environment in order to more easily compare the results. The actual amount of carcasses is not drastically larger when the thawing season is increased. As we move along the horizontal axis, we see the gap between the three lines increasing a little bit the end of each thawing cycle. We compare the three lines at around day 300, where they lie almost directly on top of each other, to day 1400 where you can see the gap enlarging. Over a decade or so, this gap would continue to get bigger. However, it's clear that the rate at which newly defrosted carcasses were introduced during the summer months made a much larger impact on the amount of carcasses seen over our time span.

The combination of both climate components can be seen in **Figure A-3**, where both the anthrax spores in the environment as well as number of carcasses are displayed. The result of a longer thawing period as well as and increase in the change in seasonality leads to a major growth of cases over the four year period. In **Figure 7**, we see this

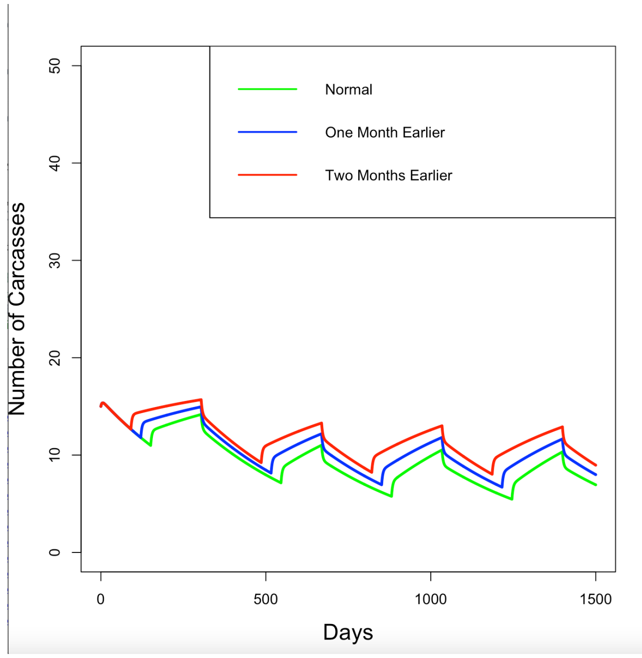

Figure 6: Longer thawing period over four years with initial conditions A(0)=20 and C(0)=15. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$, and $\gamma = .5$

conjunction again, but only on the amount of carcasses in order for the comparisons to be more legible. The green line, with no added climate effects, is following a decreasing trend. When looking at the blue line, where the active layer of permafrost is thawing one month earlier and the γ parameter is set to 1, meaning one additional carcass is defrosted each day in the summer months, the line follows more of a consistent cyclical trend. Then the red line, which has an earlier thawing start by two months and γ increased to 1.5, is following an increasing trend. We go from a decreasing trend to an increasing trend by including both climate factors. So rather than our populations declining towards the zero equilibrium, they are either remaining constant or growing, meaning more cases of anthrax will be seen. Combining both climate factors yields worse results than either one of them alone.

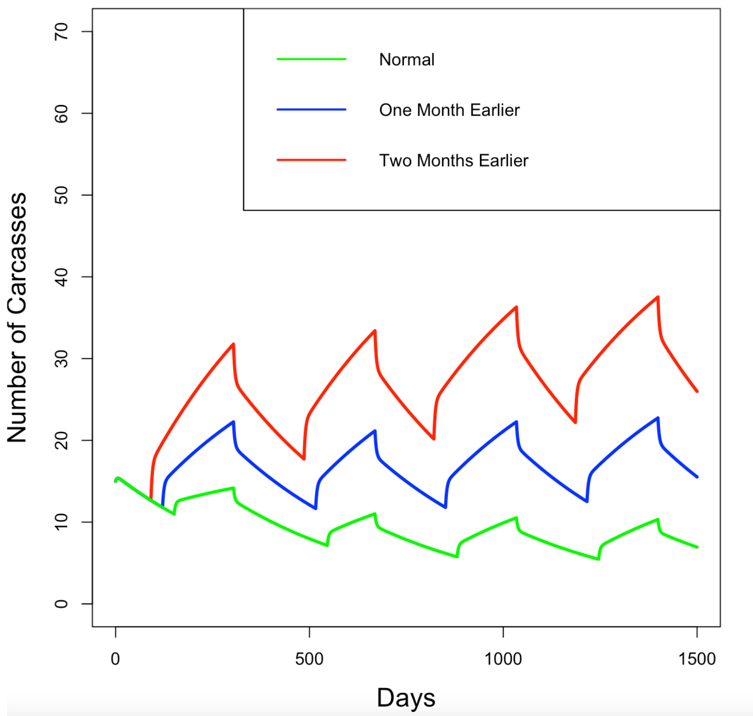

Figure 7: Longer thawing period over four years with initial conditions A(0)=20 and C(0)=15. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$, and $\gamma = .5, 1, 1.5$ for lines green, blue and red respectively

CHAPTER VI: CONCLUSION

The goal of proposing this new model was to introduce a new way of looking at the sudden increase of anthrax cases globally. Climate change has proven to have many detrimental side effects, and the release of previously frozen microbes is one of them. By taking a relatively simple model of Anthrax spread and adding global warming components, we were able to see how the change in the earth could affect Anthrax spread. Parameters for this model have been estimated using previous work on the topic. As permafrost continues to be studied, it is seen to thaw earlier and to greater depths. The change in the state of the permafrost is modelled in this paper theoretically. We observed and compared two climate changes. First we looked at what would happen if permafrost would melt to greater depths, in turn revealing more frozen carcasses. As these frozen carcasses thaw and are scavenged, anthrax spores are released into the environment around them. By increasing the amount of carcasses being exposed from the permafrost during the summer months, a huge increase was seen in the amount of anthrax cases. The other climate effect we looked into was a longer thawing period. This did not make as huge of an impact on the populations as permafrost melting to deeper depths, however we did see that there was some effect which could enlarge over decades. Through building this model, we conclude that as the Earth's surface continues to increase in temperature, the risk of Anthrax related deaths in herd animals enlarges. This puts the human population at risk of infection as well, since a lot of countries rely on herd animals to feed their populations. Humans contract the disease through ingesting contaminated meat. Thus an increase in cases among herd animals could ultimately lead to an increase in cases in humans. We learned that a greater cause of concern would be the carcasses exposing from the permafrost rather than an increase in the duration of warm months. The focus should be to eliminate the possibility of this happening. A potential solution would be to dispose of infected animals, herd or wild, through burning rather than burial. This would decrease the number of infected carcasses there are at risk of resurfacing. Though not explored in this paper, there is the potential that other diseases could reemerge from permafrost. This paper is an effort to raise questions and continue research on the harmful effects of global warming. All of the computations and models in this paper were done through R.

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APPENDIX A: ANTHRAX SPREAD IN A HERD POPULATION

Figure A-1 displays three graphs showing the effects of changing the k parameter. As stated in Chapter II, $k = .244$ is a threshold. Any value of k below .244 (seen in graph **A**) will yield results in which our populations, carcasses and anthrax spores in the environment, reach the zero equilibrium. If the parameter is set exactly to .244, the populations remain constant seen in graph **B**. Any value above .244, even just an increase of .001, leads to the populations exponentially increasing. This result can be seen in graph **C**.

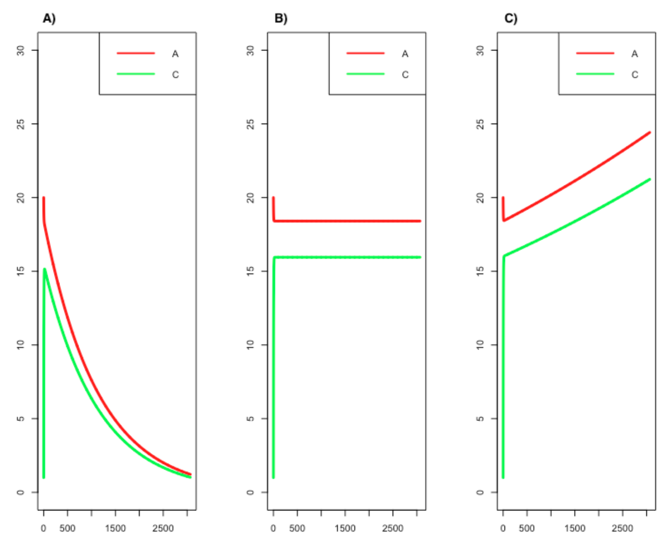

Figure A-1: Model run with initial values $A(0) = 20$ and $C(0) = 1$. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$ and $k = .235, .244, .245$ for graphs **a, b, c** respectively.

In **Figure A-2**, the result of increasing the thawing period is shown. With initial conditions and parameter values listed in the caption, we look at the outputs of the carcass values at the end of the year for each graph. We start out with 10 carcasses, and in the graph with the Normal thawing period we end with a value of 9.26 carcasses. When we increase the thawing period by starting it one month earlier, we now end with a value of 9.94 carcasses. By once again increasing the thawing period to start two months earlier, we now get an output of 10.58 carcasses. Over one years time, it is clear that there is an increase in resulting carcasses, but it may not seem that drastic of a difference. However, when looking at a four year span as we did earlier in the text, we see that this increase is continuous and the gap enlarges between the amount of carcasses with a normal thawing period and when the thawing period is extended. Over decades, this increase could be huge.

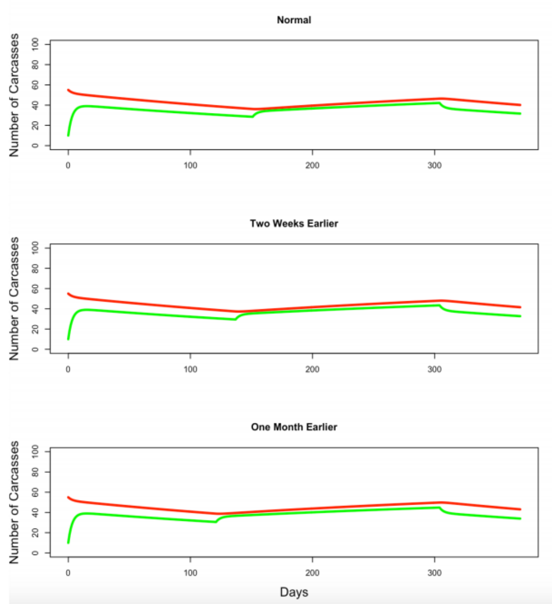

Figure A-2: Thawing of Active Layer beginning two weeks earlier with initial conditions A(0)=15 and C(0)=10. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$, and γ=.5

Figure A-3 shows the combination of both climate factors. We start with a normal thawing period and γ set to .5. Over the four years the populations in both classes rise and decline, but overall have declining populations. Then we set the γ parameter to 1 and also increase the thawing period by starting the initial melting one month earlier. There is a significant rise in both populations by the end of the four year period. The bottom graph starts the thawing period one month earlier and γ is set to 2.5. This displays an even more drastic increase than the two graphs prior. It's visible that the combination of both climate change factors yields more extreme results than either one of them alone.

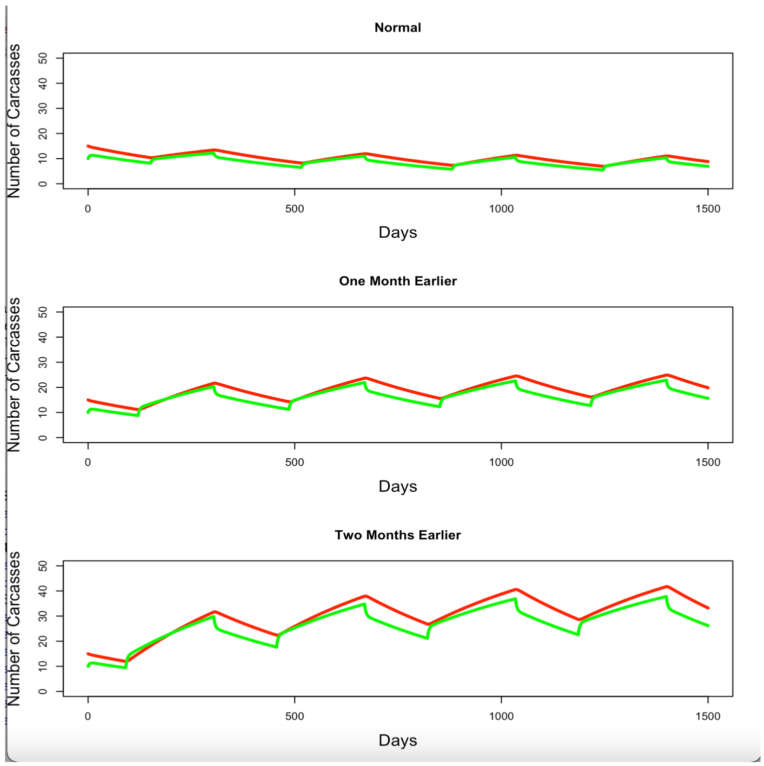

Figure A-3: Increase of thawing period over four years with initial conditions A(0)=15 and C(0)=10. Parameter values set at $\alpha = .026$, $\beta = .03$, $\delta = .2816$, $k = .2$, and γ=.5,1,1.5 for the Normal graph, One Month Earlier graph, and Two Months Earlier graph respectively.