

Nonlinear analysis for a diabetes model focus on liver handling of glucose and glucose uptake by insulin-dependent tissues

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Diabetes Mellitus is in the top ten causes of death worldwide. Over the past few years, the interest in analyzing this type of disease, either in a biological or mathematical sense, has been based on the search for a treatment that guarantees full control of glucose levels. Mathematical models inspired by natural phenomena are proposed under the prey-predator scheme and in this case as a complex relation between insulin and glucose. The analyzed model in this research is described as a fifth-order nonlinear system of ordinary differential equations that involves the variables of plasma insulin, glucose in the digestive system, regular glucose level, glucose in urine and insulin into the subcutaneous compartment. The mathematical analysis presented in this work was made by applying nonlinear control theory to define the maximum carrying capacity for each variable, establishing a bounded positively invariant domain (BPID). A particularity of the model consists in its dependency on H that is a function of glycaemia, which represents renal excretion of glucose. When plasma glucose level is lower than G_u H is zero, and there is not renal glucose excretion. When the plasma glucose level is higher than G_u , H is 1, and the renal glucose excretion depends on the difference between G and G_u . The invariant plane existence associated to insulin into the subcutaneous compartment defines a baseline to study diabetes mellitus where the model can describe a type-1 Diabetes mellitus (T1-DM) and type-2 Diabetes mellitus model (T2-DM).

Keywords: *Diabetes mellitus; Nonlinear control theory; Bounded positive invariant domain; Glucose estimation.*