ANALYSIS OF STEADY STATES FOR CLASSES OF REACTION-DIFFUSION EQUATIONS WITH U-SHAPED DENSITY DEPENDENT DISPERSAL ON THE BOUNDARY

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Abstract. We consider positive solutions to equations of the form
\[
\begin{cases}
-\Delta u = \lambda u(1 - u), & x \in \Omega, \\
\frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda}(u - A)^2 u = 0, & x \in \partial \Omega,
\end{cases}
\]
where \(\lambda > 0, \gamma > 0, A \in (0, 1)\) are parameters, \(\Omega\) is a bounded domain in \(\mathbb{R}^n; n \geq 1\) with smooth boundary \(\partial \Omega\) and \(\frac{\partial u}{\partial \eta}\) is the outward normal derivative. Such models arise in the study of population dynamics in a habitat \(\Omega\) when the population exhibits U-shaped density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness and multiplicity of positive solutions) as the patch size (\(\lambda\)) and the hostility of the outside matrix (\(\gamma\)) vary. We obtain results when \(\Omega = (0, 1)\) via a quadrature method, and when \(\Omega\) is any bounded domain in \(\mathbb{R}^n; n > 1\) by the method of sub-super solutions.

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